Axioms as models of the theorists preferences (many axioms or beliefs) which will allow him to "find" *classes* of "counterexamples" to *decision rules*, instead of stumbling across isolated "counterexamples."

Explain motivation for a) decision rules, and (b) for axioms. When will you want to improve *model*?

Distinguish (Koopman) between intuitive preferences and intuitive tests of *consistency* of judgment. Yet latter models, too, must be judged by their *consequences* (*aesthetic* considerations only *part* of the payoffs; Samuelson). Note how "subjectivist" theory leads to "objective" probs, *where appropriate*.

(Markowitz; etc; intuitive judgments about *lotteries* involving "objective" probs).

Axioms des. "coherence."

Formulate axioms of choice ⟹ Koopman/Smith behavior.

Implication of ordinal comparisons between p_i and p_j, α_i and α_j.

When *will* Sure-Thing Principle Apply?

Possibility that the simplifications and arbitrary elements in a model or rule correspond precisely (Schelling) to simplifications "we" make in *practice*, to ease calculation, speed decision.

	R	Y	B			Y	
					$0/3$	$1/3$	$2/3$
- I	1	0	0	→	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
- II	0	1	0		0	$\frac{1}{3}$	$\frac{2}{3}$
III	0	0	1		$\frac{2}{3}$	$\frac{1}{3}$	0
- IV	1	0	1		1	$\frac{2}{3}$	$\frac{1}{3}$
- V	0	1	1	→	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
VI	1	1	0		$\frac{1}{3}$	$\frac{2}{3}$	1

$$\left|\begin{array}{ccc} 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 \\ \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{2}{3} \end{array}\right.$$

$$\begin{array}{ccc} 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 \end{array}$$

$\frac{1}{9}+\frac{1}{9}$ $\sqrt{\frac{2}{9}}$ $\frac{1}{3}\sqrt{2}$

$(1,0;\frac{1}{3},\frac{2}{3})$ — — $(1,0;\frac{1}{3},\frac{2}{3})$

$(1,0;\overset{\frac{1}{3}}{0},1)$ $(1,0;\overset{\frac{1}{3}}{\frac{1}{3}},\frac{2}{3})$ $(1,0;\overset{\frac{1}{3}}{\frac{2}{3}},\frac{1}{3})$ — $\underline{(1,0;\frac{1}{3},\frac{2}{3})}$

$(1,0;1,0)$ $(1,0;\frac{2}{3},\frac{1}{3})$ $(1,0;\frac{1}{3},\frac{2}{3})$ $\underline{(1,0;\frac{2}{3},\frac{1}{3})}$

$(1,0;\frac{2}{3},\frac{1}{3})$ — — $(1,0;\frac{2}{3},\frac{1}{3})$

over

R	Y	B	
2	1	0	$\frac{5}{6}$
0	1	2	$\frac{2}{3}$
1	1	1	1
1	2	3	$\frac{5}{3}$
2	2	2	2

I have emphasized this point, because

Show how inequalities can determine decisions.

Use money for determining inequalities:

With money, Smith phenomenon ("unreasonable" by pre-Bernoulli standards)

1. McNamara's problem of making decisions. Wants to be reasonable; judge by consequences. But what will the consequences be?

B&G: "Decision theory applies to statistical problems the principle that a statistical procedure should be evaluated by its consequences in various circumstances (Neyman & Pearson, testing hypotheses; Wald, all statistical problems). Introduces considerations of values of outcomes; Savage: opinion as to probability of various outcomes (relative weight they should have, independent of value weight): decision as the resultant of these two WEIGHINGS.

R&S: values and opinions; represented by probabilities and utilities.

How to get numbers? MEASUREMENT problem; but Savage theory: measurements can be based on your own reflective choices: THEORY IS NOTHING BUT A THEORY OF CONSISTENT BEHAVIOR.

What good is a logic of your own consistent choices? a) reduce complex choices to simple ones, thus save time; b) for given decision-making time, improve decision-making; make it closer to your "best." Also, by making your preferences explicit: 1) make it possible for agent to decide; 2) expose your opinions and values to criticism, experience (improve your INFERENCES as well as your decisions), persuade others, make real differences

S 0 $-S$ 0

$p_0 S$ $p_0 S$ $p^\circ(-S)$ $p^\circ(-S)$

Requirement that $p_0 = p^\circ$ is (Savage) that there be no *kink* at 0;
"there are no clicks in Nature."

Savage: should hold if S is *small*; cont. 2d deriv.; *linear* over $(-S)$—(S)

But this assumes there is no *natural zero*. (*kink* is at $a_{11} = a_{12}$, not at $S=0$)

$-S$ 0 $\rightarrow$ 0 S

$p_0 S$ $p_0 S$

q_0 q_0

$1-p_0$ $-p_0$

0 0

$-q_0$ $1-q_0$

$-S$ 0

p^*

$p_* = \frac{1}{4}$ $p^* = \frac{1}{2}$

$-S-K$ $-K$

$p^*(-S-k)$ $p^*(-S-k)$

Imagine a *natural zero* at "no bet"; or generally, at *certain outcome*.
When probs are vague, there is "static *friction*" — to pull you off *certain* outcome, if any, positive attraction $> \epsilon$ needed.

	R	Y	B	(a)	(b)	(c)	(d)	(e)
I	1	0	0	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$
II	0	1	0		$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
III	0	0	1		$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
IV	0	1	1	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{2}{6}$	$\frac{4}{6}$	$\frac{4}{6}$
V	1	0	1		$\frac{5}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$
VI	0	1	0		$\frac{5}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$

a) Pay $\frac{1}{2} + \frac{2}{3}$ for I + II: sure to lose $\frac{1}{6}$.

b) Pay $\frac{15}{6}$ for III + IV + V: sure to lose $\frac{1}{2}$.

c) Can't make a book   e) OK

d) Can't make a book

de Finetti, etc. assume in effect (e.g. b) that pr(E) = 1 − pr($\bar{E}$).

This in itself doesn't assume additive probs in general.

Nor does coherence.

But Together they do.

So coherence requires more than $P_0 + q_0 \le 1$

also requires $\bar{P}_1 + \bar{P}_2 + \bar{P}_3 - p_1 - p_2 - p_3 \le 2$

i.e. $p(R) + pr(Y) + p(B) \le prob(R \cup Y \cup B)$

$p(Y \cup B) + p(R \cup B) + p(R \cup Y) - p(B) - p(R) - p(Y) \le$

VI   VI   VI

$p(Y) + p(B) + p(R) + p(B) + p(R) + p(Y) -$

over

INTRODUCTION

1. Action under conditions of extreme uncertainty; problem of defining reasonable behavior. Lack of objective probabilities.; or agreed "credibilities."
2. Why are we interested? ~~xxx~~ a/. Heuristic value; crude empirical theory. ~~Possibilities~~ of measurement...when relevant variables are specified.
   b. Direct interest in arriving at our own decisions; deriving complex decisions from simple ones; logic-like rules of consistency; policing our own decisions; basis for advice. Statisticians; systems ana
   c. Possibilities of measurement. Churchman.

3. Criteria of acceptable theory; test of normative axioms; Savage--will want to conform; Koopman--direct appeal to intuition; Good--doesn't force you into untenable position,need for honesty. Coherence. Intuition applied to beliefs directly: Good, Koopman.
   Axioms as a theory of one's own preferences, structure of.

Chapter I Probs not "known"; (in "objective" sense/; minimax, minimax regret, Hurwicz. Counterexamples. Axioms of complete ignorance

Chapter II

Bernoulli. hypothesis. Assumption that only preferencesand ~~xxxxxxx~~ probabilities are relevant (judgements that one event is "more likely" than another/; Ramsey. Suppes and Davidson.

If ~~xx~~ these are the only two relevant variables,then if one variable is assumed "known" or is held constant, we can infer the other from choices. Bayes: assuming that money = utility. de Finetti; likewis

Counterexamples. Bernoulli.

Or: assume that probabilities are known, measure utility; Von N-M.

Ramsey: hold utilities constant, measure prob=$\frac{1}{2}$; then measure other utilities, as in vN-M; then measure other probs.

Ramsey could have simply measured all probs as he did prob=$\frac{1}{2}$; this is essentially what Savage does; then measures utilities.

We are interested in probs, not utilities; so will use actions involving only two outcomes.

Note behavior Bernoulli hypothesis rules out --on assumption that only two variables are involved.

How is it reasonable to act when the consequences of ones actions are extremely uncertain?

Tests of normativeness: a sometimes used to mean operationally meaningful, arbitrary standards of consistency, ideal types, etc. (necessary condition for usefulness, but not sufficient
b: rules used by people;
c: lead to long-run success
d: attitudes of other people; people will say...
e: mature reflection; inner tension, inconsistency, uncomfortable... (honesty

People should act "as if" they assigned probabilities to events. But what does thismean operationally? How "does" one act when one assigns probabilities to events? Could have different behavior rules based on given states of beliefs, resulting in different behavior; there is not just one way to act when one assigns probabilities (apart from the fact that there are many ways to assign probabilities, so that any given form of "as if" behavior can lead to many different types of behavior .

E.g., given an assignment of probabilities, one could act on rule: maximize the payoff to the most probable event (act as if the most probable event were certain((needs only ordering of payoffs(

OR: ~~minimize the payoff to the~~ or: ignore probabilities under x%;

or: assign 0,1 payoffs (success, failure(and maximize prob of success, or minimize probability of failure

Or: ~~multiply~~ maximize mathematical expectation of money

Or: maximize function of math expectation of money, variance, etc.

SAVAGE RULE: act as if you maximized math expect of utility numbers, probability numbers. Implications: a(a way to measure probabilities (~~the~~ to the extent that they do follow rules
b: reasons why they ought to obey; reasonable behavior.
c: way to break down complex decisi ns into simple ones (like arithmetic

(can infer "beliefs" as to complex problems from beliefs that are relatively easy to determine, "basic."

Or: can determine "preference" tween complex alternatives by considering simplied situations in which preference is easier to determine; e.g., can determine probability for each event separately.

BUT: Savage implies that for each event you just have to ask yourself, How much would I pay for a stake of one utile on the event? THIS WILL GIVE WRONG OR IRRELEVANT ANSWERS WHEN SOME OF THE ACTIONS INVOLVE BETS AGAINST THAT EVENT, AND PROBABILITY IS AMBIGUOUS.

$$\begin{array}{cc} 1 & 0 \\ P_x & P_x \end{array}$$

$$\begin{array}{ccc} \overset{1/3}{0} & \overset{1/2}{-10} & \overset{1/6}{10} \\ \overset{1/3}{0} & \overset{1/6}{10} & \overset{1/2}{-10} \\ \overset{1/3}{-10} & \overset{1}{0} & \overset{1/2}{20} \\ -10 & 0 & -20 \\ 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccc} 0 & 0 & 1 & \frac{1}{6} \\ 0 & 0 & -1 & -\frac{1}{2} \\ 1 & 0 & 0 & \frac{1}{3} \\ -1 & 0 & 0 & -\frac{1}{3} \\ 0 & 1 & 0 & \frac{1}{6} \end{array}$$

$\frac{10}{3} - \frac{10}{2} = \frac{20-30}{6}$

$\frac{20}{6} - \frac{30}{6}$ $\frac{10}{6}$

$\frac{-20}{6} + \frac{20}{6} = 0$ $\frac{20}{6} - \frac{60}{6} = -\frac{40}{6}$

over

1. Book extends Mosteller-Nogee experiment; uses experimentally determined subjective prob ($\frac{1}{2}$) instead of objective probs, complicated bets.

2. Thus, where M-N took their model from von Neumann-Morgenstern (through Friedman-Savage), Suppes-Davidson go back to Ramsey.

3. They found event Ramsey called for: ZEJ, WUH.

4. They found events equally spaced in utility--in sense that if outcomes are ordered a,b,c, indifferent between b and aXc.

5. For these scale is derived; predictions made about other bets.

6. Then prob of other events can be measured.

7. ~~But~~ Results are not strikingly good. They are perhaps better than predicting max of math expect of money, or simply observing ordering, or flipping a coin--but it is not surprising that we should predict risk-choices better on the basis of other risk-choices than we can do without any observations of risk-choices.

8. What are the nature of the deviations? Hint: 2 didn't want to play; were they minimaxing? But no analysis of actual deviations.

9. But then, Suppes has no hypothesis about possible deviation, even of a negative sort, e.g., that a particular axiom is being violated. Nor do they ever discuss different ways in which we might predict from risk-choices, different patterns of behavior; in fact, they never spell out just what kind of behavior would violate the axioms, let alone when it is most likely.

10. What is the role of the axioms? They may serve to increase the plausibility of some proposition--e.g., that numbers can be assigned to outcomes and events--hence recommend the proposition for testing; or they may recommend themselves as guides to behavior, from which numbers could be derived. But they are not expressed so ~~tha~~ as to persuade us that people do or should obey them--unlike the Savage axioms . Anyway, in an empirical test, these aspects would be irrelevant.

Axioms might still be useful to classify possible areas of violation. But they make no attempt to relate violations to axioms; e.g., to say that a particular one was most frequently violated.

So what are the axioms good for?

11. Even if results were perfect, what would we have? Utility measurements on sums less than a dollar--no basis for extrapolation, and unlikely that they could be extrapolated; prob measurements. Latter might be interesting--could be any events (though perhaps less likely to be as successful with other events).

They didn't explore interesting result that prob was less than 1/4; what about fact that same subjects union of two of these events as equal to $\frac{1}{2}$.

12. Nice to have philosophers and psychologists worrying about our problems

13. no attempt to predict prefs or probs from non-risk data--e.g., from risk-data compiled from other persons similar in some respects.

14. does not investigate how subjective probs are derived from observable variables--e.g. objective frequencies; hence, can't derive probs for events other than the experimental events.

15. In other words, tests very limited set of hypotheses about behavior; not necessarily those most interesting, accesible to test, or plausible; simply the ones suggested by the Ramsey-Savage-approach.

16. Choices with different prizes can't be predicted with this experimental setup; differences between classes of people could be tested, but real differences will probably only show up with large sums of money; but relation of probs to observable variables could be investigated, even with small prizes--then inferences could be made about probs of unobserved events. So experimental setup seems better suited to studying prob.

17. Reason that they don't investigate how people do form subjective probs (i.e., how they arrive at a priori dist; how to predict dist) is that there seems ~~to~~ no way to prescribe their attitudes--rational norms don't apply. Yet this will be the most interesting Question: not how to measure or represent expectations (which is important; this book examines one possibility), but, for a given method of measurement, how to predict, explain, expectations. Personality, class, role, may give a clue; or individual betting behavior may remain the best clue--but even so, betting behavior need not reveal probabilities, butsome different "measure" of likelihood.

Given I > II, III > IV, V ≐ VI, prove VII > VIII

VIII	0	100	0	100	0	100
VII	100	0	0	0	100	100

suppose VII = VIII

100	0	0	0	0	0	
0	100	0	0	0	0	
100	0	0	100	0	100	P2
0	100	0	100	0	100	
100	0	0	100	0	100	P1
100	0	0	0	100	100	
0	0	0	100	0	100	P2
0	0	0	0	100	100	

cont.

suppose VIII > VII

100	0	0	0	0	0	
0	100	0	0	0	0	
100	0	0	0	100	100	P2
0	100	0	0	100	100	
0	100	0	100	0	100	P1
0	100	0	0	100	100	
0	0	0	100	0	100	P2
0	0	0	0	100	100	

cont.

VIII Knight: confidence. Keynes: weight of evidence. But: how to represent "low confidence," "low weight of evidence," "vagueness"; and how to act on it?

~~Relation of p_*, p^* to p~~ Define p_*, p^*; Y^0 Diagram: degree of ambiguity

Good, on intervals of probs: VAGUES OF TYPE II, III PROBS, ETC.: FISHER
CONFLICTING FORECASTERS ! RULES AS "SUPER-CRITERIA" to resolve conflicts

Issue is: How do you act when you have wide intervals of probs?
SOMETIMES IT DOESN'T MATTER (Savage, R&S, de F): BUT WHAT IF IT DOES?

Violate P1
Pick one distribution: (Schlaifer, R&S, ~~Jeffre~~ , de Finetti, Knight?, Savage)

or randomly (de F, SMITH..

or assign equal probs (Jeffreys); but R&S criticism, troubles with assigning any one distribution; do you really accept all implications? Try it out on 0, 1,2 Yellow High odds bets ~~ELLSBERG~~

MY RULE: gives meaning and impact to "confidence," judgment;

Violates axioms, but: does it conform to your behavior? Compare it to alternatives;

Relation of "upper and lower betting probs" to "upper and lower intuitive probs."

Good comment. Observe bets on + bets against. Both should be in Y^0
p_0 p^0

Maximizing lower prob

Hypothetical explanations of rule; ~~eff~~ in terms of example;

Consequences; conservatism of minimax version, incoherence of maximax;

value of info, insurance. Shackle and Keynes on effect of ambiguity/low confidence

Raiffa comments on "importance": my reply.

Trouble with minimax, etc. ("no" opinions): lower betting prob "too low"; not in Y^0: $\alpha_0 - 1_0$ upper probs "too high": make you pass up good bets
(My rule doesn't)

#

Uncertainty in Economics, G.L.S. Shackle, Cambridge, 1955

"The application of frequency-ratios only makes sense if the individual can feel sure that there will be many repetitions, that there will be a divisible experim in which the immediate act, with which is now concerned will be swallowed up." 7

crucial experiments "Now it is plain, I think, that for a non-divisible non-seriable experiment no frequency-ratio can have any meaning or relevance." 7 "The fact that out of 600 throws a die has shown an ace 100 times tells us that out of 6000 throws it will show an ace about 1000 times. But what does it tell us about the next throw, the 601st? Suppose the captains in a Test Match have agreed that instead of tossing a coin for choice of innings they will decide the matter by this next throw of ~~a die~~ the die, and that if it shows an ace Australia shall bat first, if any other number, then England shall bat first. Can we now give any meaningful answer whatever to the question 'Who will bat first?' except 'We do not know'?" ~~8~~ For a non-divisible non-seriable experiment the concept of frequency-ratios is wholly irrelevant." 8

"What basis is there left on which a person, faced with a set of rival courses of action each of which is a non-divisible non-seriable experiment, can make his choice? If we ask what in such a case it is rational to do there is no answer, if rationality means choosing the most preferred amongst a set of attainable ends. For he does not know what ends are attainable with the means at his disposal, indeed, in the face of this ignorance, his powers of action are not properly described as 'means' at all." 9

"When an experiment is such thatits result will be obtained by adding together the results of a series of separate performances, and when these performances are going to be sufficient uniform in their circumstances, and sufficiently numberous, for frequency-ratios derived from a past series of similar performances to be applied, I shall call the experiment, which consists in the totality of the contemplated series of performances considered all together as one whole a divisible experiment." 5 or if experiment will be repeated many times; a seriable trial.

Doesn't like numbers "like" frequency-ratios as measures of plausibility, because they don't convey the right "feel" intuitively to him. 27.

But has a point: in my terms, it is possible for one "betting ratio" to go down or up without others changing.

"notion of degrees of disbelief, of doubt, rejection, or implausibility...By disbelief I do not now mean the absence of perfect certainty, but the positive recognition of some disabling circumstance. If I am shown a closed hat-box and asked to guess what kind of hat is within, I shall attach no disbelief, in the sense in which I am now using the word, to the idea that the hat may be a topper, a ten-gallon hat from Texas, or a mortar-board, but I shall attach a high degree of disbelief to the hypothesis that the hat is of the three-foot diameter kind which ladies used to call a picture-hat: because such a hat would not go in the box; or at any rate, would not go in without being crumpled in a way that conflicts with my experience of how such hats are treated by their owners." 30

for word "disbelief," he then substitutes "potential surprise." "absence of disbelief"= "a state of mind such that, if the person concerned were suddenly to obtain knowledge of the matter and find that the hypothesis were in fact true, he would feel no mental discomfort of shock, he would not be in the least surprised." 30

34 he indicates he would assign zero potential surprise to all the different types of moderate-sized hat he could think of.

If players in a game don't know the payoffs with certaihnty, "would the von Neumann and Morgenstern hypothesis be more plausible than that the decision-maker will choose that action whose best possible outcome is the best amongst the respective best possible outcomes of all the actions open to him?" 43
more plausible hypothesis is "thathe will take into account both the 'best possible' and the 'worst possible' outcome of each course of actionand make these pairs of outcomes the basis of his decision."

See, H.R. Parker, Some Notes on Risk, Metroeconomica, vol v 1953, 138-144

counter-expected event vs. unexpected (unanticipated, unthought-of) event. 57

exhaustive set of rival hypotheses (I feel certain true answer will be among them); vs. residual hypothesis, recognition of non-exhaustiveness (might have zero surprise; i.e., I expect to be surprised; character of the true hypothesis, in detail, will be surprising to me.)
((e.g., explanation of Soviet current posture))

VII

"it may well happen that not one of the more adventurous of the courses of action open to him escapes the need of a residual hypothesis to render exhaustive his set of rival hypotheses concerning its outcome. If he attaches low or zero potential surprise to these residual hypotheses, all the points representing active and 'positive' policies may lie below the origin indifference-curve, and only those representing passive policies, typically that of keeping his fortune in cash, may lie on or above it. Thus we should expect that when the news seems to provide internally inconsistent or conflicting evidence about the policies or intentions of powerful persons, or concerning the degree of knowledge they possess, or is for any reason especially difficult to interpret, the consequence would be to inhibit some kinds of business activity; not because the news was regarded as bad from the point of view of those engaged in this activity (often they are divided into two camps with opposing momentary interests, like the Stock Exchange bulls and bears), but because it is unintelligible. And this we qctullly observe to happen." 61-62.
(see Economica, vol. xx, 1953, 112-17: Logic of Surprise
((Like Keynes, General Theory. People prefer not to bet, at certain odds, in a certain state of ignorance.))

crucial trial...one whose outcome, like a chess-move, willaffect the whole future course of relevant events for the individual. "It is because I believemost occasions of choosing are, and are felt to be, crucial that I attach so much more importance to the unique or isolated experiment than to the kind typified by drawing balls from an urn or tossing a coin (except when the latter is for choice of stations in the Boat Race or of innings in a Test Match)." 63 see above for his views on Test Match innings.
"But my belief in the focus-values solution, as opposed to the integrative solution, depends really on how I conceive the nature of expectation itself, as an act of creative imagination and not of "rational" calculation; for calculation is impossible when the data are incomplete, and in face of ignorance, rationality is a mere pretence." 74

HYPOTHESES ARE MEANINGFUL, CONTRADICTORY TO SAVAGE, AND SOMETIMES TRUE (Arrow theorem, Hurwicz rule; or my rule): SET WITH ZERO POTENTIAL SURPRISE, AS DEFINED BY BEHAVIOR SHACKLE POSTULATES CAN EXIST. HIS MISTAKE IS TO ARGUE THAT THIS IS TRUE IN ALL SITUATIONS OF CRUCIAL CHOICE: OVER-EXTENDS APPLICATION OF NOTION.

Precision in the expression/measurement of our beliefs + preferences may come from the application of precise rules we construct for ourselves.

Goals of some: a) to generate prior probs logically, necessarily, deductively;
b) to generate "objective" probs on which all "reasonable" men must agree.
c) to ensure that precise probs can be assigned to all events.

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1. Bernoulli approach implies that people act "as if" they had definite opinions; i.e., assigned definite probability numbers to every event.
2. Theorem: if they don't, they violate axioms they won't want to violate: Savage... certain canons I want never to transgress. SIMPLE FORM.
3. Could "test" axioms; but disproof of one set doesn't prove another won't work. On the other hand, the theorem can be tested directly; has empirical content.
4. Consider urn example. It violates Postulate 2. Rationale.
5. Evidence that people (me, Raiffa's examples, Chipman's experiment, Becker experiment, want to violate); then Bernoulli proposition neither describes, nor is acceptable theory of our behavior; we don't act "as if" we assigned definite probs and used Bernoulli proposition; and if we measured definite probs by some method and applied them, we would get answers we wouldn't want to live by.

For the moment, simply consider this example as indicating that: a) Bernoulli theory has content, it can be violated; (b) there is some reason to believe that it may be violated; (c) troubles seem to arise, in this case, in connection with "vagueness," indefiniteness of probs--though this is not associated with indefiniteness of preferences. If a different theory is needed, it is "probably" a theory that explains definite preferences in the presence of "ambiguity." ((Imagine contradictory information on urn.

6. Savage himself indicates problems with vagueness; but hard to talk about it in his theory. We need way to measure it, discover it, express it; then think about how one might act with it.
7. ~~COMPARE~~ Bernoulli-Bayes-Ramsey-de Finetti-Savage line does make the notion of definite opinion meaningful, show how to measure it. We can "measure" or put limits on vague opinions by comparison with definite opinions . Let us make some hypothetical measurements, and then examine critically the notion that all opinions "should" be measurable (implication of axioms).

NECESSARY CONDITION FOR QUALITATIVE PROBS: THERE MUST BE SOME NUMERICAL PROBS (NOT TYPICALLY UNIQUE) THAT "AGREES WITH" CHOICES.

A) Raiffa-Schlaiffer: if you want to behave consistently with axioms/Bernoulli prop, measure your own expectations and apply probabilities. What if you don't know your own mind? They show how to measure your probabilities by asking yourself how you would bet on simple alternatives. This will result in definite, precise probs--if you obey axioms. Let us suppose that you do obey axioms w.r.t. at least some events; WE NEEDN'T PREJUDGE QUESTION OF WHETHER YOU ALWAYS OBEY AXIOMS, HENCE, WHETHER YOU CAN MEASURE PROBS FOR ALL EVENTS; IN ORDER TO MEASURE MEANINGFUL PROBS FOR SOME EVENTS, YOU DON'T NEED TO OBEY AXIOMS W.R.T. ALL EVENTS.

establish inequalities; — then unique probs.

Methods like Bayes, de Finetti, Ramsey, late Savage, can generate definite probs (when utility is linear on money), if person obeys appropriately; however, he can disobey without violating Savage axioms (for large sums; Bernoulli examples). Odds.

Implications of Savage axioms on: how much you will pay for bets.

Next chapter: my examples; show violation of Bernoulli principle. Show violation of Savage axioms.

Show failure of Borel comparisons to attach definite probs; but do indicate limits on probabilities

Smith argument applied to my example

Criticize coherence assumption (I don't buy it, applied to betting probs; assume either tat you know betting probs and decision rule--deduce intuitive probs; or assume that you know intuitive probs and decision rule--deduce betting probs; or assume you know intuitive probs and betting probs--deduce decision rule. But if decision rule is close to maximax, you don't get betting "probs".

1. Savage (late) approach: probs as prices of contingencies.

2. But: utility problems; Bernoulli examples. If utility not linear on money, you get exactly Smith behavior.

3. But we have already measured utility: Ramsey method (Bernoulli example).

4. Even when utilities are substituted (or when utility is linear, or when we restrict ourselves to small bets) SMITH PHENOMENON is meaningful (not ruled out by general assumptions on utility; only by assumption on -S, etc.)

5. Ramsey on consistency and coherence, w.r.t. utility. BUT CRITICISE COHERENCE ASSUMPTION, IN TERMS OF UTILITIES.

7. Smith examples ~~give meaning~~ (Fellner also) give meaning to vague probs; show that you can construct a theory; partial ordering; comparisons of upper and lower; medial odds; can predict behavior (in betting on or against given event). But: (a) is it reasonable? It does violate axioms. (b) ~~Is it~~ How do you decide among actions with many consequences? Randomly?

6. Show whole thing in terms of odds; utility problem; requirement of reciprocals.

8. Compare "two-price system" to Savage notion of "price."

9. Savage on "vagueness" of prices; compare Smith: p_o and p^o may be hazy, but say for $10 on E I may feel sure that 5$ is too much; and for $10 on $\bar{E}$ I may feel that 10$ is too much. SAVAGE, ETC: FOR A GIVEN STAKE, E.G., $10 ON, no

Chapter 4:

1. Savage axiom of inference.

2. Three-horse race.

3. To ensure choices are possible: P3, P4, P5.... To ensure enough numerical probs to compare to, P6.

4. Important: P1 and P2.

5. Borel comparisons. Relation to FELLNER.

6. How Smith phenomenon turns up in Borel comparisons: p_o, p^o (not necessarily the same as before?). Show violation of P2 (Also, show violation of former Smith phenomenon of P2)

knocks out coherence. Y^o convex.

HYPOTHESIS: REASONABLE BEHAVIOR REQUIRES THAT CHOSEN ACTION HAVE HIGHEST EXPECTATION FOR SOME SET OF REASONABLE PROBABILITIES (GOOD). (BUT LOOK AT WHOLE SET OF EXPECTATIONS

The Consequences of Vagueness for Choice

0	−10	10	−2M	+1M	+1M
0	10	−10			
−10	0	20			
10	0	−20			

Denote by $A_{\frac{1}{100}}$, $A_{\frac{2}{100}}$ $A_{\frac{3}{100}}$ *appraisable* events such that prob$(A_{i/n}) = \frac{i}{n}$

A_i	E	$\bar{A}_i \cap \bar{E}$	$A_i \cap E$
1	0	0	1
0	1	0	1
1	0	1	0
0	1	1	0

H	T	E	$\bar{E}$	E	$\bar{E}$
10	0	10	0	10	0
0	10	0	10	4	4
5	5			−10	0
−10	0			−6	−6
−5	5				

E	$\bar{E}$	E	$\bar{E}$
1	0	0	1
p_*	p_*	q_*	q_*

Axioms: $p_* + q_* = 1$

Coherence

Me: $p_* + q_* \leq 1$

	E	$\bar{E}$
I	2	-1
II	0	0
III	-2	1

Savage: Can't have both II > I

and II > III

II > I $\quad p \cdot 2 + (1-p)(-1) < 0$

$$3p < 1$$

$$\underline{p < \tfrac{1}{3}}$$

II > III $\quad -2p + (1-p) \cdot 1 < 0$

$$-3p < -1$$

$$p > \tfrac{1}{3}$$

Suppose $p_* + q_* < 1$ Let p' be such that $p_* < p' < p^*$

$q_* < (1-p') < q^*$

Bet	$(1-p')$	$-p'$	Bet	$-p'$	$1-p'$
Don't	0	0		0	0

Prefer not to bet, either at $\frac{p'}{1-p'} : 1$ on E or against E at $\frac{p'}{1-p'} : 1$

e.g. if $p_* = \frac{1}{4}$, $q_* = \frac{1}{4}$, $p' = \frac{1}{2}$ I won't bet at 1:1 on E or against E.

Insurance action: not betting

$$\begin{array}{cc} 1 & -1 \\ -1 & 1 \\ \rightarrow 0 & 0 \end{array}$$

I will not pay $(\frac{1}{2})$ for $\begin{array}{cc} \overline{E} & \\ 1 & 0 \end{array}$

nor $(1-\frac{1}{2})$ for $\begin{array}{cc} 0 & 1 \end{array}$

(Savage: for any x $0 \le x \le 1$, either $\left(\begin{array}{cc} 1 & 0 \\ x & x \end{array}\right.$ or $\begin{array}{cc} 1 & 0 \\ 1-x & 1-x \end{array}$

$$\begin{array}{cc} \overset{25}{R} \cup \overset{25}{Y} \cup \overset{25}{B} & \overset{25}{W} \\ 10 & -30 \\ 0 & 0 \end{array} \qquad \begin{array}{cc} R \cup Y \cup B & W \\ 10 & -30 \\ 0 & 0 \end{array}$$

$$\begin{array}{cc} E & \overline{E} \\ 1 & -2 \\ 0 & 0 \\ -1 & 2 \end{array}$$

Arrow conditions for complete ignorance. Me: may assign $< \frac{1}{n}$ prob_* to every event.

UNUSED QUOTES:

Schlaifer: practical method for making decisions. may be difficult. v.

R&S: logical consistency requires...

Keynes: so far I yield to Ramsey...

Ramsey: probability of 2/3 is equal to two out of three propositions (further reflections

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	Heaven	Hell
Life	0	0
Death	1000	-5000
L	1000	0
D.	0	5000
(5/6, 2;)	833	833

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IX Keynes, General Theory – betting on contests (confidence)

Fisher, bets on higher ranks

Jeffreys, vagueness of upper ranks

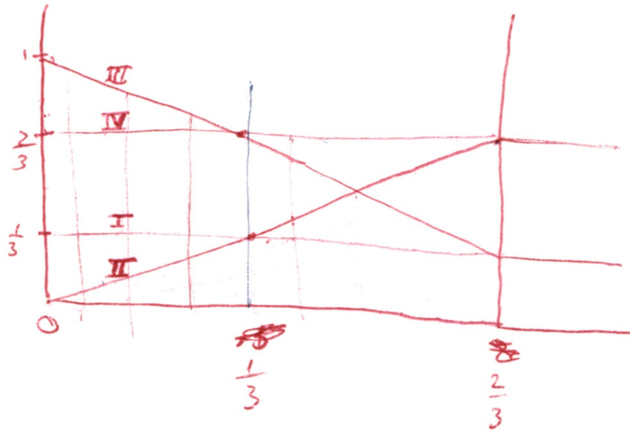
Savage: vagueness + minimax

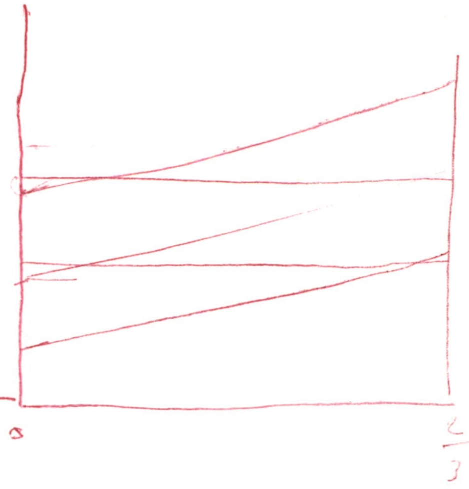
Rubin
Milnor
Allais

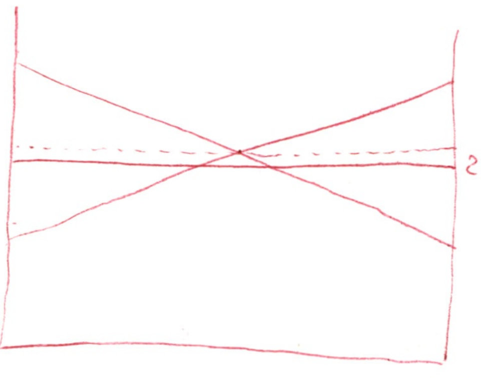
No?

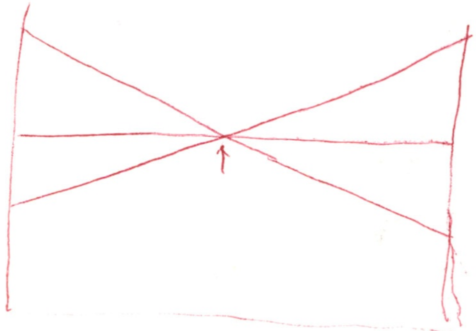
Will maximax violate rule that it must be a Bayes' sol. for some reasonable y?

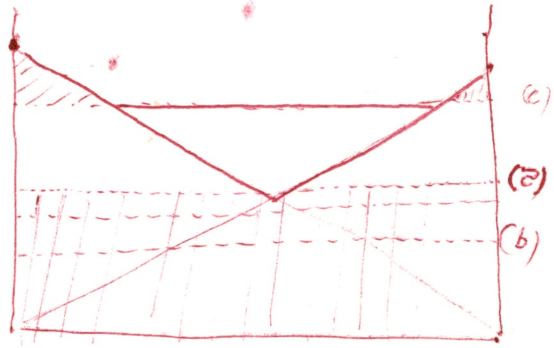
Must rule out $\alpha = 0$ and $\alpha = 1$ (or have lexicographic utility); otherwise, violate admissibility).









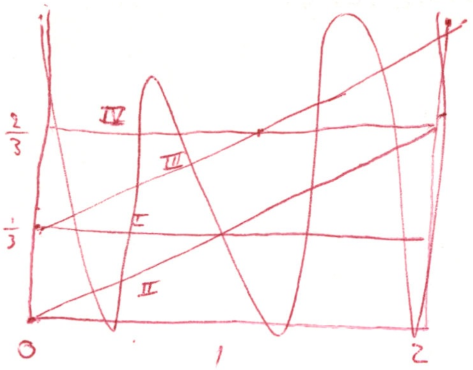


Hyp: there must be some reasonable dist. for which chosen strategy is among the optimal set. i.e. must be some dist for which expected regret would be 0.

(No given dist. need explain whole ordering of strats

Is (a) the "most expensive acceptable insurance"? Is (b) "too expensive"—unacceptable?

Discard everything which is not a Bayes' solution for some reasonable dist (this will usually exclude minimax).



R	Y	B
1	0	0
0	1	0
0	0	1
1	0	1
0	1	1
1	1	0

(1 0 | 0
0 1 | 1
(0 1 | 0
1 0 | 1
(0 | 0 1
1 | 1 0

Y

	0/3	1/3	2/3
I	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
II	0	$\frac{1}{3}$	$\frac{2}{3}$
III	$\frac{2}{3}$	$\frac{1}{3}$	0
IV	1	$\frac{2}{3}$	$\frac{1}{3}$
V	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
VI	$\frac{1}{3}$	$\frac{2}{3}$	1

1 2 3
2 2 2

R	Y	B
2	1	0
0	1	2
1	1	1
3	2	1
2	2	2

1 3
2^- 2^-
3 1

0 10
9 9

10 0 -10
0 0 0

10 0
9 9

1 3
2^- 2^-
3^{--} 1

100
190
280

300
190
100

170

200

1

$\frac{-9}{10}$

R	Y	B	
1	0	0	$\frac{1}{3}$
0	0	1	$\frac{1}{6}$
10	0	0	$\frac{10}{3}$
0	0	20	$\frac{10}{3}$
-10	0	20	0
-1	-1	-1	
0	0	0	0
10	0	-20	$-\frac{40}{6}$ ←
20	0	-20	$-\frac{20}{6}$
20	0	-15	$-\frac{5}{6}$
15	0	-10	0 ←
~~-10~~	~~$\overset{1/2}{-10}$~~	~~$\overset{1/2}{20}$~~	~~-5~~
~~0~~	~~$\overset{1/2}{-10}$~~	~~$\overset{0}{0}$~~	~~-5~~
-10	$\overset{1/2}{10}$	20	5
0	$\overset{1/6}{10}$	0	$\frac{10}{6}$
10	10	0	5
0	10	20	$8\frac{1}{3}$

10 0 -10

$\frac{20}{6} - 3$

$\frac{40}{6} - \frac{45}{6} = -\frac{5}{6}$

$\frac{30}{6} - \frac{3}{6}$

VII

$\frac{1}{3}$	$\frac{1}{6}-\frac{1}{2}$	$\frac{1}{6}-\frac{1}{2}$	
R	Y	B	
1	0	0	$\frac{1}{3}$
0	1	0	$\frac{1}{6}$
0	0	1	$\frac{1}{6}$
0	1	1	$\frac{2}{3}$
1	0	1	$\frac{1}{2}$
1	1	0	$\frac{1}{2}$

I	0	0	0
II	100	0	-200
	-2	1	1
	0	1	0
	0	0	1

SAVAGE ALSO IMPLIES: AT MOST, YOU NEED TO MEASURE PROBS OF ~~NXXX~~
N-1 EVENTS.
DON'T NEED TO MEASURE PROBS OFEVENTS WHICH HAVE
CONSTANT PAYOFFS

N

VI

must work harder and "make up their minds" on a specific probability distribution, or select one randomly, or pretend that they have a specific distribution in mind: so as to reach a decision, by applying a sort of decision rules available. (See Schlaiffer to this effect.) The available rules which did not require precise specification of probabilities -- for instance, minimax, minimax regrets, or the Hurwicz criterion -- seemed designed only for very special cases of "total ignorance"; if one had at least some knowledge of probabilities, no matter how vague or incomplete, these rules could be made to look foolish with respect to particular problems. In other words, to avoid looking foolish (in terms of or of intuition the Savage axioms) when they had any knowledge at all of relative likelihoods, decision-makers had to select, or act "as if" they had selected, a specific probability distribution; and they had to do this independently of the payoffs in any particular problem. The decision-makers were left with no way left to express or act on a judgment × that it was hard to decide within a broad range of distributions in some cases, and harder in some cases than others. Yet such judgments seem often highly relevant to dedision, and to go far beyond minor problems of "precision".

Good expanded the comments he had made on the imprecision of probability judgments in "rational decisions" in: "Discussion on the ~~Supposium~~ Symposium on Linear Programming" Journal of the Royal Statistical Society; Series B, Vol. 17, No. 2, 1955, pp. 195-196. Here he sets out to define the "type II minimax rule"; he considers decisions of the type of accepting H_1, H_2, H_3, with ~~psi~~ ξ being a type II hypothesis (possibly subjective) that asserts initial probabilties over the hypotheses. "It is typical of subjective probabilities that they are not usually judged to have precise numerical values, but are judged to lie in wide or narrow intervals of values. This fact can be expressed by saying that there are a number of different type II hypotheses, ~~psi~~ ξ, all of which you regard as

× if they should act, e.g. by H-L or my rule, they would be "foolish" by S's axioms; S implies they should act by, e.g. random or arbitrary selection.

R	Y	B	
1	0	0	p_*
0	1	0	r_*

1 0 0 0 1 1

0 1 0

$1-p$ $-p$ $-p$ $-q$ $1-q$ $1-q$

1 0 0 1

p_* p_* q_* q_*

$p_* + q_* \leq 1$

$r_* + s_* \leq 1$

$p_* < r_* \Leftrightarrow s_* < q_*$

me: $p_* < r_* \Leftrightarrow \overset{s^*}{(1-r_*)} < \overset{q^*}{(1-p_*)}$

$s_* < q_*''$

$q_* \leq 1 - p_*$

$s_* \leq 1 - r_*$ p

$p_* = r_* \Leftrightarrow s_* \leq q_*$

$p_* > r_*$? $(1-r_*) > (1-p_*)$

$p_* >$

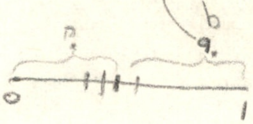
s_* a a a

I a q_* a

II b b

a a

b



B2: $p_* > r_* \Leftrightarrow s_* > q_*$

$=$ $=$

$<$ $<$

Let $p^* = 1 - q_*$, $r^* = 1 - s_*$

Savage:

$p_* = 1 - q_* = p^*$

Coherence:

$-p_*$

$-p_*$ $-r_*$ $\bar{A}A\bar{B}$

R Y B $1-p_*$ $1-r_*$ $1-s_*$ $\bar{A}$ B 0

0 $-r_*$ $-s_*$ $1-q_*$ 0 0_{p_*}

Rather bet on Red than on Yellow; and rather bet against Red than against Yellow.

I 1 0 p_* $1-s_*$ $1-q_*$ 1 0 p_* 1

p_* p_* 0 0 0 0

1 $-q_*$ 0 p_* 1 1 q_*

II 0 r_* 1 0 q_* p^*

r_* 0 0 1 0 q_* p^*

0 s_*

III 1 s_* 1 q_* p^*

s_* 1 q_*

0 q_*

IV q_*

$I : II \Rightarrow III \boxed{\leq} IV$

$I < II \Rightarrow III < IV$

But $I > II = III \lesseqgtr IV$

1. Point is not the symmetry of information (or ignorance) as it is the low information; not complete ignorance, but considerable ignorance.

2. e.g., example of man who prefer either side of 60:40 bet to either side of 6:4.

3. Set of prob distributions; may be able to assign probability weights to these (e.g., equal); but question of confidence about these. Where confidence is low, expectations based on this composite probability dist may not seem like adequate basis for decision.

4. This doesn't contradict payoff numbers assigned, or indicate new ones. Those numbers reflected fact that in another context, this same probability distribution was held with higher confidence, and expectations computed on the basis of it and assigned payoff numbers did correspond to preferences.

Assume outcomes of $100, $0, $40, and utilities of ~~10, 0,~~ 1, 0, .5, reflecting the fact that the subject was indifferent between staking $100 on either Heads or Tails on the toss of a coin (justifying prob of .5 for that event) and was indifferent between having (paying) $40 ~~for~~ or a $100 bet on Heads. But although he is equally willing to bet on Red or Black in the unknown urn, he will pay only $30 for the bet.

5. Confidence, credibility, ambiguity: same concept as Georgescu-Roegen; but ~~his~~ his assumption that ordering of uncertainties is lexicographic is implausible (and contradicted by those subjects who still prefer R_{II} when a Red ball is removed from the ~~Red urn~~ known urn) (Though G-R denies that probability can be compensated by credibility, subjects who prefer RII will not do so if the difference in relative probabilities is made sufficiently unfavorable to RII.) His ~~assump~~ assertion that lexicographic ordering would make it impossibl to measure uncertainty dissolves if we limit set of uncertainties to a countable infinity.

6. Different levels of ambiguity. Axioms rule out that subject should prefer to stake a given prize on A than on B, and also on A' than on B'; hence, rule out that subject should pay more to bet on A than on A', and also more to bet on A' than on B'.

6. Judgement of ambiguity, credibility, confidence, cannot be expressed in terms of relative likelihoods or weights. Not a matter of the relative support the evidence gives to a particular hypothesis over another, but the question of how much weight the evidence should give any hypothesis

7. Effects of rule; conservatism (2 senses); favors actions less sensitive to ambiguity.

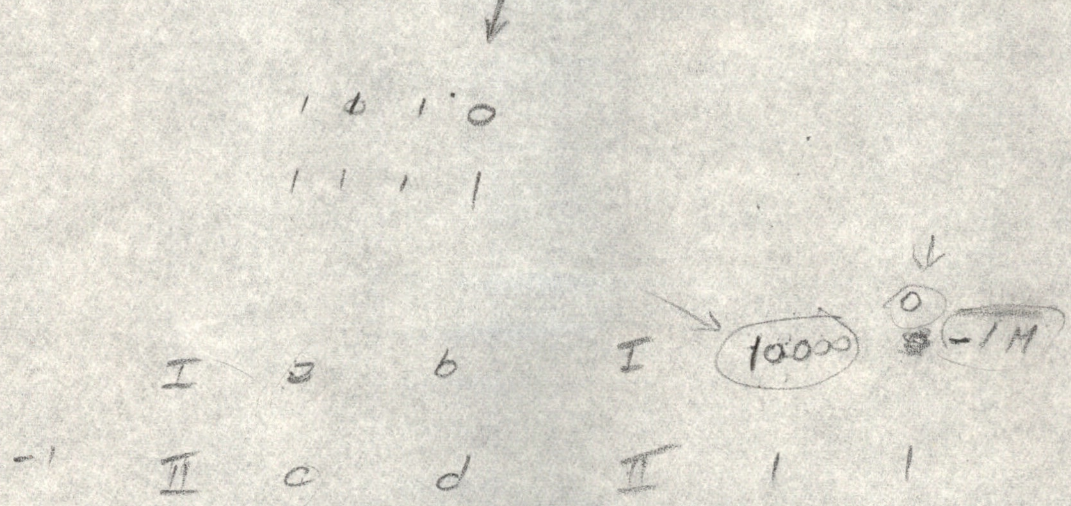
8. $\rho = 0$ Hurwicz
9. H+L

Scientists are interested in "hypotheses" that a) would "make a difference" if true, and (b) have, in their minds, a certain minimum chance of being true and provable; this threshold scale depending on 1) the difference it would make, and 2) the cost of proving

---

My rule as sub-optimization.

How do we survive with such crude systems analyses? Partly because SU has same problems.



Shackle

Like S, my decision rule implies: the amount I may pay to bet on E may go up without the amount I will pay to bet against E going down; "revealed probs" of E and not-E can vary independently (or, upper and lower probs can vary independently).

New hypotheses can be formed by splitting up the residual hypothesis without need of lowering betting probs on other, initially explicit hypotheses. p. 10

crucial experiment

Shackle asks: What numbers properly "convey" my "true state of mind"?

	R	$\bar{R}$
Obey C	0	-100
Obey H	-100	1000

10 0

$\frac{1}{10}$

$\frac{1}{4}$

$\frac{3}{4}(1) + \frac{1}{4}\cdot 5 = R$

$\frac{3}{4}\cdot 2\frac{1}{2} + \frac{1}{4}\cdot 5$

Insurance: Because of decreasing MU, people are willing to make unfair money bets; insurance company is thus able to make a book against the field; or at least, to make so many favorable bets it is "almost sure" it will win (its assets may also be so large MU is linear). When bet is ambiguous, insurance company demands odds that are "too favorable"; it passes up opportunities to "almost make a book."

$\rho = \frac{9}{10},\ \alpha = \frac{1}{4}$

$\frac{9}{10}\cdot\left(\frac{1}{10}\cdot 10\right) + \frac{1}{10}\left(\frac{1}{4}\cdot 10 + \frac{3}{4}\cdot 0\right)$

$.9 \qquad .225 = 1.125$

Even so, cost of insuring against failure may be so high that it also insures against success.

$\frac{1}{10}\cdot x = .225$

$x = 2.25 \qquad x = \frac{1}{4}$

Breakeven point for Hurwicz rule:

$P(\text{max payoff}) = \alpha$.

$\frac{9}{10}\left(\frac{1}{4}\cdot 10\right) + \frac{1}{10}\left(\frac{1}{4}\cdot 10\right)$

e.g. $\alpha = \frac{1}{4}$ explains Preston-Baratta results.

for any ρ.

$\frac{1}{2}$

Thus, small sure-thing is worth less than ambiguous risk with same MEU (P-B experiment: same MEM); just as more ambiguous risk > less ambiguous, bad risk "known" (~~either~~ through prior observation of betting behavior or by some other ~~operation~~ process)

P4 is equivalent to saying:

I	1	0
II	0	1

your preference between I and II should not be affected by replacing the outcome a represented by 1 by some other outcome a' ($a' > b$).

Savage: recognizes "pull" of examples; must be dealt with, analyzed (also sees Allais' examples). but axioms are good enough basis for statistics (esp. given observations; then neither differences in probs or initial ambiguity matter much. BUT HOW ABOUT DECISIONS UNDER IGNORANCE: when info is not cheap, or available at all.)

Savage: People used to think maximizing expected utility in terms of cash was obvious and inevitable.

assumption that if

I	1	0
II	p_*	p_*
III	2	0
IV	$2p_*$	$2p_*$
V	-1	0
VI	$-p_*$	$-p_*$

I = II, then III = IV, has same effect of assuming linear utility, as assumption that if I = II, then V = VI.

These all used to be regarded as self-evident for money (Bayes, de Finetti).

1	0	0	1
p_*	p_*	q_*	q_*

Reasonably

	E	$\bar{E}$
I	1	0
II	p_*	$\bar{p}_*$
III	0	1
IV	$\bar{q}_*$	q_*

$p_* + q_* \leq 1$

$p^* = 1 - q_*$

$$\begin{Bmatrix} -1 & 0 \\ -p_* & p_* \end{Bmatrix}$$

$(1 - p_*)$ $(-p_*)$

First Anscombe – Dept. of Math, Princeton,

Instead of

If utility if concex, then if prob of E is ½, ~~utility of prize is xx~~ I will bid, say, only $4o for a prize of $1oo on E, and only $40 for a prize of $100 against E; so I am coherent, but will pass up a chance to make a book against an opponent. If utility is concave, then I may bid $60 (utility = ½(on E and $60 against E; again I am coherent, but a book can be made against me in money. Where probabilities are definite, so $p^* = p_*$, the principle of coherence "makes sense"--i.e., avoids these two possibilities--only if utility is linear on money. Otherwise, if utility is convex, I should be allowed to accept unfair bets in order to make a book, and if utility is concave, I should reject fair utility bets--insisting on favorable utility bets--to avoid a book being made against me in terms of money.

A *dominated* system may be desirable to retain as possibility, as insurance if *objectives* may change.

If we progressively *discard* dominated strats, we may not reconsider when payoffs change.

Even *maximaxer* doesn't violate *Koopman* axioms of belief (and obey *Good* criterion: uses a *reasonable* dist always) even though he is incoherent.

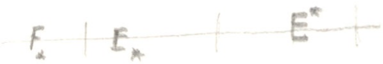
CHAPTER VI

Is the behavior examined in Chapters IV and V to be regarded as reasonable? By this point we have seen a good deal of evidence in favor of accepting it as such; yet certain objections remain, to be considered in this chapter and the next.

In favor ~~its~~ of its reasonablness are such arguments as the following. A significant number (perhaps a majority) of people otherwise considered reasonable exhibit these patterns of behavior, under circumstances that can often be identified or predicted on objective evidence. These are people who in other identifiable circumstances do obey the Savage axioms. At worst, then, they are not always "irrational"; ~~and~~ (And even if the behavior in question were regarded as unreasonable, an explanation such as that proposed in the preceding chapter could still be regarded as a valuable descriptive theory, a "theory of errors," if you like, essential for predicting their behavior in circumstances when "unreasonable behavior is to be expected").

	E		
I	1	0	0
II	0	1	0

$I > II \Rightarrow E_* > F_*$

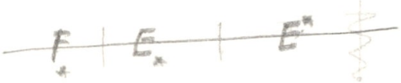


	E	F	
III	1	0	1
IV	0	1	1

$III > IV \Rightarrow E_* < F_*$

$(1 - E^*) < (1 - F^*)$

$E^* > F^*$



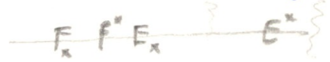
	E^*	F_*	
V	-1	1	0
VI	1 E_*	-1 F^*	0
VII	0	0	0

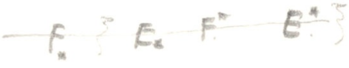
$VI > VII \Rightarrow F^* < E_*$

$VI < VII \Rightarrow F^* > E_*$

$VI > V$

non-overlapping





E^*		
-1	0	0
0	-1 F^*	0

Shoffner: α may depend on Y_0 & $\alpha > \frac{1}{2}$ if $\text{prob}(\text{win})^\circ < \frac{1}{2}$.

P2 merely rules out one interval *enclosing* another

E_*		
1	1	0
1	0 F^*	1

Insuff. Reason: *designates* n-scales. Koopman, etc: *assumes existence* of n-scales, exploits those it finds.

Minimax: When two actions have same outcomes, neither one *dominates*, must be *indifferent*.

Minimax regret: same

1	0	1
0	1	1

Hurwicz: same

Shackle: same

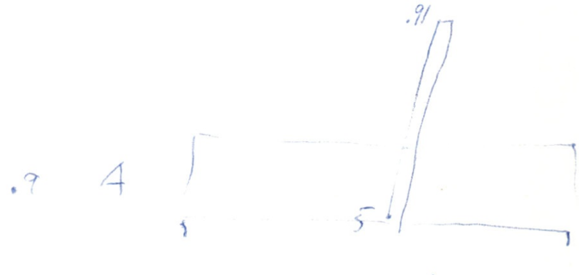
394-5184

Why ~~not~~ max moral expect. . .

$$\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & \\ 1 & 0 & 1 & \end{array} \qquad \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{array}$$

value of value of

invariance vs. flexibility: ambiguity favors toughening?



$.9(A\ B\ \Sigma) + .1(A, B, C \ldots F)$

fuzziness about ends of intervals

CP1–

flex: may "expect" greater uncertainty later; (with possibility of less)
not certain of now
if certain. . .

is minimax action 'reasonable'?

$p^*(E) \geq p_*(F)$

~~$p^*(\bar{E})$~~ $p_*(\bar{E}) \leq p^*(\bar{F}) \rightarrow (1 - p^*(E)) \leq (1 - p_*(F))$

$p^*(E) > p_*(F)$

~~$(1 - p^*(E))$~~

$p^{*}_{*}(\bar{E}) > p^* \bar{E})$

I	1	0	0
II	0	1	0

I > II $\quad p_*(E) > p_*(F)$ $\quad (1 - p^*(\bar{E})) > (1 - p^*(\bar{F}))$

$p^*(\bar{E}) < p^*(\bar{F})$

III > IV

III	1	0	1
IV	0	1	1

$p^*_*(E) > p_*(\bar{E})$ $\quad (1 - p_*(F)) > (1 - p^*(E))$

$p_*(F) < p^*(E)$

$p_*(\bar{F}) = (1 - p^*(F)) > (1 - p^*(E)) = p_*(\bar{E})$

$p_*(F$

Prove $p_*(E) > p^*(F)$

or $p_*(\bar{F}) > p^*(\bar{E})$

1. Decision-maker needs to know extent of "reasonable ambiguity."
Since even reasonable individuals vary in their ability to "perceive,"
recognize, report ambiguity, the "committee" or panel of isolated
experts or several groups of interacting experts may be way to get true "feel" of
range of reasonable opinion — if they can be induced to report their
differences as well as their consensus (one problem: they might thereby lose
chance to "coerce" decision by their concerted opinion: JCS problem).
Also, their opinions must be comparable; otherwise, differences in reporting
might exaggerate real difference or real conformity. Also, if probs expressed
as odds, odds "will reflect their utilities.

(USIB. NORAD: should two commanders report interpretation
of BMEWS? Post-attack: decisions through consensus.).

2. In C & C: crucial importance of the costs of info, etc., on the rigidity &
content of plans. ~~St~~ Plausible to assert that technological factors alone
compel form of response, "regardless" of values put on alternative
strategic objectives. ; or to assert that costs of achieving flexibility (at least,
and true
as (over-) estimated by CINCS/ are such as to make strategic objectives
critical to the design of the system — if indeed one believed that there were
alternatives to the deterrence-only objectives.

Homeostasis of strategic objectives: ~~A~~ self-maintaining aspect
of objectives, plans, technology, operations, organization, information/intelligence,
R & D, budget, enemy posture, mutual beliefs. (Boulding). (and expectations).
NOT INERTIA; dynamic resistance to change of
(Cognitive dissonance).
objectives, plans, strategy. (Other things change to maintain

1. Prob. defined by statements or intuitions need not (will not) correspond to probs defined by acts.

2. When are probs comparable?

3. What can you infer from acts about probs?

4. Suppose you will pay more for bet on A than on B, and on $\bar{B}$ than on $\bar{A}$.

5. [illegible] Whether you ever obey axioms despite internal estimates of probs depends on whether you ever

(1136 Embury

Laplace rule when money amounts are small?)

RADCLIFFE CLUB OF SOUTHERN
438 SOUTH OGDEN STR
LOS ANGELES 36, CALIFO

John Pratt

Like Shapley, Pratt stresses the "game" aspects of the situation as presented. Necessary assumption; that the person offering you the bets doesn't know any more than you do. Hence, his offers give you no information about the ~~xxx~~ contents of the urn. (An experimenter may have chosen the contents non-randomly, but he doesn't control the bets offered you, nor does he stand to gain or lose by the outcomes of the bets. The person offering the bets may be interested in measuring your subjective probabilities, or he may just be interesting in betting--without having any inside info, though his probabilities may well be different from yours (assume that if you knew his, it wouldn't change yours; knowing the bets that are acceptable to him doesn't affect the bets that are acceptable to you).

Should be easy to imagine this; like bets on an election, or the likelihood of war, or the average rainfall in Uganda, or which drop will slide down the windshield first: with your neighbor.

Pratt asks: Suppose you know, or believe, that the person offering you the bet chose randomly between offering you a bet on Red or a bet on Black. This obviously doesn't affect your information about the contents of the urn. ~~Exx~~ Does it affect your betting?

NO.

	50	50	100	
	R_I	B_I	R_{II}	B_{II}
I	1	0	0	0
II	0	1	0	0
III	0	0	1	0
IV	0	0	0	1

to I or II.

Suppose I prefer ~~I to~~ III or IV. If I know you are going to toss a coin between offering me III or offering me IV, ~~Ixhaxe~~ and will offer me I as an alternative in either case, I have ~~fxxx~~3strategies beforehand:

EV

1) Take I if I offered, III otherwise; $\frac{1}{2} \cdot u(I) + \frac{1}{2} \cdot \frac{1}{2}$ $\frac{1}{4} - \frac{3}{4}$

2) Take II if II offered, III otherwise $\frac{1}{2} \cdot u(II) + \frac{1}{2} \cdot \frac{1}{2}$ $\frac{1}{4} - \frac{3}{4}$

3) Take III in either case $\frac{1}{2}$ $\frac{1}{2}$

I am indifferent between the first and second, but I prefer the third to either. The expected value payoff, the distribution over payoffs, is still ambiguous in the first two cases; the probability of 1 may range from 1/4 to 3/4 (likewise, the expected value), whereas the probability of payoff 1 is definitely 1/2 in III.

If I do not attach definite probabilities to your offering me one bet or the other--in particular, if I find this just as ambiguous as the outcome of the bet--then I will give even less for strategies (1) and (2); the probability of a payoff of 1 now can range from 0 to 1.

If, however, you offer me a choice between taking III or flipping a coin between I and II, I will be indifferent .

Decision Processes
edited by Thrall, Coombs and Davis
John Wiley and Sons, New York, 1954

Introduction, by Davis:

"For instance, the theory of zero-sum two-person games can be taken as normative theory in the sense that it gives instructions according to which a man will be able to maximize his expected payoff in such a game, assuming he can find the solution. But this does not say anyone should use this theory in playing an actual game: it may be that he can more easily secure this maximum expectedpayoff insome other way, or it may be that he is an iconoclastwho sees no sufficient connection between mthematical expectation and the outcome of any play of the game to justify the procedure." p. 5

((The first statement is false, by the usual meaning of the term "expected payoff." The second confirms this interpretationof expected as math. expect, and suggests that the only reason for rejecting has to do with repeated plays and aversion to the math expect)

Chap. 4, GamesAgainst Nature, John Milnor

p. 56 S Implies that the best criterion would be one that satisfied certain desirable logical properties; actually, the Wald criterion, say, might not be desirable evenif it did satisfy all the given requirements.

$$\begin{array}{cccc}0&0&0&0\\0&0&2&-1\end{array}$$

$$\begin{array}{cccc}2&2&0&0\\2&2&2&-1\end{array}$$

$$\begin{array}{cccc}0&0&2&2\\2&2&2&-1\end{array}$$

$$\begin{array}{cccc}0&0&-1&2\\2&2&-1&-1\end{array}$$

$$\begin{array}{cccc}0&0&-1&2\\-1&-1&2&2\end{array}$$

$$\begin{array}{cccc}0&0&-1&-1\\-1&-1&2&-1\end{array}$$

$$\begin{array}{cccc}-1&-1&0&0\\-1&-1&2&-1\end{array}$$

$$\begin{array}{cccc}0&0&0&0\\0&0&2&-1\end{array}$$

Suppose $\swarrow\begin{array}{cccc}0&0&0&0\\0&0&2&-1\end{array}$ and $\swarrow\begin{array}{cccc}0&0&-2&-1\\0&0&2&-1\end{array}$

Prove $\swarrow\begin{array}{cccc}0&0&-2&1\\0&0&0&0\end{array}$

Let $\swarrow\begin{array}{cccc}0&0&0&0\\0&0&-2&1\end{array}$ and show contradiction

---

$$\swarrow\begin{array}{cccc}0&0&-2&1\\0&0&2&-1\end{array}$$

$$\begin{array}{cccc}0&0&0&0\\0&0&-2&1\end{array}\quad P1$$

and $\downarrow\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 \end{array}$

Show this leads either to:

$\downarrow\begin{array}{cccc} 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}$ or $\downarrow\begin{array}{cccc} 0 & 0 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{array}$

by evolution of (a)

$$\downarrow\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 \end{array} \qquad \begin{array}{cccc} -2 & -2 & 2 & 2 \\ 0 & 0 & 2 & -1 \end{array}$$

$$\begin{array}{cccc} 1 & 1 & -1 & -1 \\ 0 & 0 & 2 & -1 \end{array} \; P1 \qquad \begin{array}{cccc} -2 & -2 & -1 & 2 \\ 0 & 0 & -1 & -1 \end{array}$$

$$\begin{array}{cccc} 1 & 1 & -1 & 2 \\ 0 & 0 & 2 & 2 \end{array} \; P2 \qquad \begin{array}{cccc} -2 & -2 & -1 & 2 \\ -1 & -1 & 0 & 0 \end{array}$$

$$\begin{array}{cccc} 1 & 1 & -1 & 2 \\ 1 & 1 & 1 & 1 \end{array} \; P1$$

$$\begin{array}{cccc} 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 \end{array} \; P2$$

0

Suppose $\begin{matrix} 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$ Suppose $\begin{matrix} 0 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 \end{matrix}$

and $\begin{matrix} 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{matrix}$ and $\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 \end{matrix}$

Show: either ~~2 -1 0 0~~ $\begin{matrix} 0 & 0 & 2 & \\ 0 & 0 & 0 & 0 \end{matrix}$

or 0

E

Not both $\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 \end{matrix}$

and $\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 \end{matrix}$

$\begin{matrix} a & b \\ b & a \\ 0 & 0 \\ a & a \\ 0 & 0 \\ b & b \end{matrix}$

Suppose both of above; show contradiction

$a > b > c$

$\begin{matrix} a & b \\ b & a \\ c & c \end{matrix}$ when $\begin{matrix} a & a \\ b & b \\ c & c \end{matrix}$

if a, b, c are utilities

Not $\begin{matrix} c & c \\ a & b \end{matrix}$ and $\begin{matrix} c & c \\ b & a \end{matrix}$ or $\begin{matrix} a & b \\ c & c \end{matrix}$ and $\begin{matrix} b & a \\ c & c \end{matrix}$

Suppose $\swarrow\begin{array}{cccc}0&0&0&0\\0&0&2&-1\end{array}$ Show $\swarrow\begin{array}{cccc}0&0&-2&1\\0&0&0&0\end{array}$

and $\swarrow\begin{array}{cccc}0&0&-2&1\\0&0&2&-1\end{array}$ necessary?

Let $\swarrow\begin{array}{cccc}0&0&0&0\\0&0&-2&1\end{array}$ and show contradiction, e.g. $\begin{array}{cccc}0&0&-2&1\\0&0&0&0\end{array}$, or $\begin{array}{cccc}0&0&2&-1\\0&0&0&0\end{array}$

or $\begin{array}{cccc}0&0&-2&-1\\0&0&2&-1\end{array}$

$$\swarrow\begin{array}{cccc}0&0&0&0\\0&0&-2&1\end{array}$$

$$\swarrow\begin{array}{cccc}0&0&0&0\\0&0&2&-1\end{array}\quad P1$$

$$\begin{array}{cccc}1&1&-1&-1\\0&0&2&-1\end{array}\quad P1$$

$$\begin{array}{cccc}1&1&-1&2\\0&0&2&2\end{array}\quad P2$$

$$\begin{array}{cccc}1&1&-1&2\\1&1&1&1\end{array}\quad P1$$

$$\begin{array}{cccc}-1&-1&-1&2\\-1&-1&1&1\end{array}\quad P2$$

$$\begin{array}{cccc}-1&-1&-1&2\\0&0&0&0\end{array}\quad P1$$

$$\begin{array}{cccc}-1&-1&-1&2\\-2&-2&2&2\end{array}\quad P1$$

$$\begin{array}{cccc}-1&-1&-1&-1\\-2&-2&2&-1\end{array}\quad P2$$

$$\begin{array}{cccc}-2&-2&0&0\\-2&-2&2&-1\end{array}\quad P1$$

$$\swarrow\begin{array}{cccc}0&0&0&0\\0&0&2&-1\end{array}\quad P2$$

-2 -1 0 1 2

Use
$$\left(\begin{matrix} 2 & 2 & -1 & -1 \\ 0 & 0 & 1 & 1 \\ -1 & -1 & 2 & 2 \\ 1 & 1 & 0 & 0 \end{matrix}\right. \quad \left(\frac{1}{2}\right)$$

$$\left(\begin{matrix} -2 & -2 & 1 & 1 \\ -1 & -1 & 0 & 0 \\ 1 & 1 & -2 & -2 \\ 0 & 0 & -1 & -1 \end{matrix}\right. \quad \left(-\frac{1}{2}\right)$$

$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 \end{matrix} \longleftarrow \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 \end{matrix} \qquad \begin{matrix} -2 & -2 & 2 & 2 \\ 0 & 0 & 2 & -1 \end{matrix}$$

$$\begin{matrix} 2 & 2 & 0 & 0 \\ 2 & 2 & 2 & -1 \end{matrix}$$

$$\begin{matrix} 0 & 0 & 2 & 2 \\ 2 & 2 & 2 & -1 \end{matrix}$$

$$\begin{matrix} 0 & 0 & -1 & 2 \\ 2 & 2 & -1 & -1 \end{matrix}$$

$$\begin{matrix} 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 \end{matrix}$$

$$\begin{matrix} -2 & -2 & -1 & 2 \\ -2 & -2 & 1 & 1 \end{matrix}$$

$$\begin{matrix} -2 & -2 & -1 & 2 \\ 0 & 0 & -1 & -1 \end{matrix}$$

$$-2 \quad \overline{-1 \quad 0} \quad 1 \quad 2$$

What is greatest $w \ni I \geq II$? $\left(w = \frac{p}{1-p}\right)$

$$\begin{array}{lcc} I & S & -wS \\ II & 0 & 0 \end{array}$$

w = l.u.b of w's $\ni I \geq II$

e.g. $w =$

e.g. if $w = 3:1$ then if $w = 3$

$$\begin{array}{lcc} I & S & -3S \\ II & 0 & 0 \end{array}$$

$I = II$

if $S = 1$,

$$\begin{array}{lcc} I = & 1 & -3 \\ & 0 & 0 \end{array}$$

Smith: If Umpire announces 3:1 on B against C, that means Bob must be prepared to ***give*** betting odds 3:1 on B if he is to bet at all **on B**

i.e. ~~give~~ get 1:3 ~~[illegible]~~ on B

$1 \quad -3$

Smith calls this "accepting" odds of 3:1 on B.

1   0   .25   -1   0   0   1

$p_*(1)$ $p_*(1)$ $p_*'(-1)$ $p_*'(-1)$ $q_*(1)$ $q_*(1)$

$q_* = p_*' + 1$ $p_* \cdot (-1) + 1 = q_*(1)$

$p_*' = q_* - 1$ ~~q_*~~ $=$ $p_* = 1 - q_*$

S   0   -S   0

p_*^S p_*S $p_*(-S)$ $p_*(-S)$

$p = (\frac{1}{4} -)$

$$\begin{array}{lll} I & 10 & 0 \\ II & \frac{1}{4}\cdot 10 & \frac{1}{4}\cdot 10 \end{array} = \begin{array}{ll} 10 & 0 \\ 2.5 & 2.5 \end{array} \qquad \begin{array}{llll} I & -10 & 0 & -2.5 \\ II & \frac{1}{4}\cdot -10 & -2.5 & -1 \end{array}$$

smaller p, ($p = \frac{1}{8}$), I will prefer I ; larger p, I will prefer II

$$\begin{array}{lll} I & 10 & 0 \\ II & p\cdot 10 & p\cdot 10 \end{array}$$

What is largest p for which $I > II$?

say, $p = \frac{1}{4}$.

$p_* = \frac{1}{4}$

E

$$\begin{array}{llll} I & -10 & 0 & -2.5 \quad -.5 \\ II & p\cdot(-10) & p(-10) & \end{array}$$

What is largest p for which $I < II$?

$I: -2.5$

$p = \frac{1}{4}$, $I = II$

$p = \frac{1}{8}$, $I < II$

$p = \frac{3}{4}$, $I > II$

$I: (-2.5) - (-.5)$

$p \overset{?}{=} \frac{1}{2}$

~~$p_*(S) \le p_*(-S) = q_*(S)$~~

$p_*(E)^S \le p_*(E, -S) = \overset{1-}{q}$

$I \quad S \quad 0 \qquad I' = -S \quad 0 \qquad I'' \quad 0 \quad S$

II $v(I'') = v(I') + S$ or $q = 1 - p$ or $p = 1 - q$

$v(II'')$ $q_*(S) = v(II') + S$ so if $v(II') = p_*(-S)^{+S}$, $q_*(S) = p_*(-S) + S$

$-S \quad 0$

$p(-S) \quad p(-S)$

$\text{I} \quad -S - p(-S) \quad -p(-S) \quad -$

$\text{II} \quad 0 \quad 0$

pS

$p_*(S) \le p^*(-S)$

$\text{I} \quad -S(1-p) \quad pS$

$0 \quad 0$

What is largest p for which $\text{I} \le \text{II}$?

$\text{I} \quad S \quad 0$ What is greatest $p \ni v(\text{I}) \ge v(\text{II})$?

$\text{II} \quad p_*(S) \quad p_*(S)$

$\text{I}' \quad 0 \quad S$

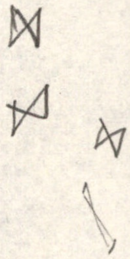
$\text{II}' \quad q_*(S) \quad q_*(S)$ " " " $q \ni v(\text{I}') \ge v(\text{II}')$?

$\text{I}'' \quad -S \quad 0$ What is greatest $p \ni v(\text{I}') \le v(\text{II}')$?

$\text{II}'' \quad p^*(-S) \quad p^*(-S)$

$v(\text{I}') = v(\text{I}') + S$ | $q_*(S) = p^*(-S) + S$ | $q_* = 1 - p^*$

$p^* = 1 - q_*$



$$p_*(E, S) \leq p_*(E, -S) = (1 - q_*(E, -S) \cdot p_*(\bar{E}, S)$$

$$\begin{array}{cc} S & 0 \end{array}$$

$$\begin{array}{cc} (S - p_* S) & -pS \\ 0 & 0 \end{array}$$

$$\begin{array}{c|cc} & p & 1-p \\ \hline \mathrm{I} & (1-p_*)S & -pS \\ \mathrm{II} & 0 & 0 \end{array}$$

p_* is l.u.b. of p's ∋ $\mathrm{I} \geq \mathrm{II}$ $\qquad \omega = \frac{p_*}{1-p_*}$

divide by $(1-p_*)$

↑

$$\begin{array}{ccc} \mathrm{I}' & S & -\omega S \\ \mathrm{II} & 0 & 0 \end{array}$$

$p_* \leq p$

$$p(1-p_*)S + (1-p)(-p_* S) = 0$$

$$p(1-p_*)\cancel{S} - p_*(1-p)\cancel{S} = 0 \qquad \frac{p}{1-p} = \frac{p_*}{1-p_*}$$

I am going to discuss the following problem. Given various circumstances, to decide what to do. ~~Gi~~ My main contention is that our methods of making rational decisions should not depend on whether we are statisticians.

There may be occasions when it is best to behave irrationally, but whether there are should be decided rationally.

Clearly I am sticking my neck out in discussing this subject. In most subjects people usually try to understand what other people mean, but in philosophy and near-philosophy they do not usually try so hard. 107

I.J.Good, Rational
Decisions
(Jour. Royal Stat Soc
No. 1, 1952)

The obsession with objectivity arises largely from the desire to be convincing in published work. There are, however, several activities in which it is less important to convince other people than to find out the truth for oneself. 109

Numerical probabilities can be introduced by imagining perfect packs of cards perfectly shuffled, or infinite sequences of trials under essentially similar conditions. Both methods are idealizations, and there is very little to choose between them. It is a matter of taste: that is why there is so much argument about it. 110

What has made the confidence interval popular is "indicating what information is available." Decision problems seem beside this point; a confidence interval probably contains the parameter, and the confidence level measures how probably. But does it? By the formal definition, it no longer does, once we insert numerical values for the endpoints. Then no probability (except 0 or 1) can be attached to the event that the interval contains the parameter; either it does or it doesn't. Unfortunately we don't know which. We think, and would like to say, it 'probably' does; we can invent something else to say, but nothing else to think.

John W. Pratt, review of Testing Statistical Hypotheses by E.L. Lehmann, Jour of the Am Stat Assoc Vol 56, No.293, March 1961, p. 165

a b b b

b b a b

A

I > II ⇒

	A	C	B	D
I	1	0	0	0
II	0	0	1	0
	0	1	0	0
	0	0	0	1
	1	0	1	0

A	B	$\overline{A\cup B}$
1	0	0
0	1	0

10 0

0 10

a b

1 0

7 9

1	0	0
0	1	0
1	0	1
0	1	1

A		B
1	1	0
0	1	1
10	10	5
5	10	10

$\overline{B}$		$\overline{A}$
1	1	0
0	1	0

A		B	$\overline{A\cup B}$
5	^{3}5	3	3
3	^{3}5	5	3
9	9	7	7
7	9	9	7

A		B	$\overline{A\cup B}$
1	1	0	0
0	1	1	0

$\overline{B}$

	$A\cap B$	$\overline{A}$	
1	1	0	0
0	1	1	0

R_{I}	B_{I}	R_{II}	B_{II}
1	0	0	1
0	0	1	1
	↑		↑

M E M O R A N D U M

To: All at RAND - Santa Monica   Date: 1-3-62

From: Dave Lederer   Memo No.: M-40

Subject: 1962 AUTOMOBILE LICENSE PLATES

RAND will pick up your 1962 automobile license plates from the Motor Vehicle Department beginning Monday, January 8 through Friday, February 2.

The guard in the South Lobby will accept your 1962 registration card together with cash in the correct amount or a check made out to cash in the amount shown on your card. If you pay by check, please have your address on the check.

Please sign your registration card and stub on the reverse side in the place indicated.

If you have not received your 1962 registration, the guard can obtain your 1962 plates from the 1961 registration which is probably on the steering post of your car. He can obtain the cost of your 1962 plates by checking on the charge for 1961.

The penalty for not having your license plates by the prescribed time (Midnight, February 5, 1962) is 100% of the registration fee ($8.00) and 50% of the license fee. For example, if your registration tab reads $40.00 the following penalties would apply:

$ 8.00	Normal registration fee
8.00	Penalty for late registration (100% of $8.00)
32.00	Normal license fee
16.00	Penalty for late license fee (50% of $32.00)
$64.00	TOTAL

Plates cannot be obtained for trailers, trucks or out-of-state automobiles.

Dave

Dave Lederer

DAL:js

fill in:

Keynes' change in views

Good's axioms, implying numerical values for probs

~~Carnx~~

Systems analysis as "intelligence," Marschak, p. 189.

Quote Marschak, three lectures, p. 176; postulates "intuitively appealing"

In my example, note that if we had *just* learned, e.g. that R_I ~ B_I, we would have concluded $p(R_I) = p(B_I) = \frac{1}{2}$ (Ramsey). Need more choices to disprove axioms.

e.g. if we asked: $(R_I, B_I; \frac{1}{2}) > R_I, B_I$

Violates

Why would person *prefer* to flip a coin (which Raiffa seems to imply!)
(Do his students change their original preferences in *my* problem?)

Because now he *knows* prob (prize) is precisely $\frac{1}{2}$; which he *doesn't* know for pure strat.

Pratt/Raiffa emphasize that $(R_I, B_I; \frac{1}{2})$ "should" $= (R_{II}, B_{II}; \frac{1}{2})$
True! It does! They overlook that $(R_I, B_I; \frac{1}{2})$ is *preferred* to R_I, or B_I! Mixed strat is *different*; doesn't just "put the problem in a new light."

Marschak refers to alternative geometries (181, Three Lectures) but doesn't really admit alternative axioms of choice. 188

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189. Game Theory assumes symmetry of behavior norms.

Savage theory allows for quite varied behavior; *allows many types earlier theories ruled out* (includes them as *special cases*. *Yet* it may not permit some reasonable behavior, and it may require some *unreasonable* behavior.

Examples (It *does* exclude e.g. minimax; but that looks *unreasonable* in many cases; whereas Bayesian behavior "rarely" does — till my examples).

RABIES VACCINE

a b b b

b b a b

A

I > II ⇒

	A	C	B	D
I	1	0	0	0
II	0	0	1	0
	0	1	0	0
	0	0	0	1
	1	0	1	0

A	B	$\overline{A\cup B}$
1	0	0
0	1	0

10 0

0 10

a b

1 0

7 9

1	0	0
0	1	0
1	0	1
0	1	1

A		B
1	1	0
0	1	1
10	10	5
5	10	10

$\overline{B}$		$\overline{A}$
1	1	0
0	1	0

A		B	$\overline{A\cup B}$
5	3 5	3	3
3	3 5	5	3
9	9	7	7
7	9	9	7

A		B	$\overline{A\cup B}$
1	1	0	0
0	1	1	0

$\overline{B}$

	$A\cap B$	$\overline{A}$	
1	1	0	0
0	1	1	0

R_{I}	B_{I}	R_{II}	B_{II}
1	0	0	1
0	0	1	1
	↑		↑

M E M O R A N D U M

To: All at RAND - Santa Monica
Date: 1-3-62

From: Dave Lederer
Memo No.: M-40

Subject: 1962 AUTOMOBILE LICENSE PLATES

RAND will pick up your 1962 automobile license plates from the Motor Vehicle Department beginning Monday, January 8 through Friday, February 2.

The guard in the South Lobby will accept your 1962 registration card together with cash in the correct amount or a check made out to cash in the amount shown on your card. If you pay by check, please have your address on the check.

Please sign your registration card and stub on the reverse side in the place indicated.

If you have not received your 1962 registration, the guard can obtain your 1962 plates from the 1961 registration which is probably on the steering post of your car. He can obtain the cost of your 1962 plates by checking on the charge for 1961.

The penalty for not having your license plates by the prescribed time (Midnight, February 5, 1962) is 100% of the registration fee ($8.00) and 50% of the license fee. For example, if your registration tab reads $40.00 the following penalties would apply:

$ 8.00	Normal registration fee
8.00	Penalty for late registration (100% of $8.00)
32.00	Normal license fee
16.00	Penalty for late license fee (50% of $32.00)
$64.00	TOTAL

Plates cannot be obtained for trailers, trucks or out-of-state automobiles.

Dave

Dave Lederer

DAL:js

1. I wish to talk precisely about vagueness: how to identify it, and describe it, and measure it; how it may affect decisions; and how it is reasonable to act on the basis of vague knowledge.

2. If you obey the Bernoulli criterion--if you obey the Savage axioms--we can infer a probability distribution for you over all events. We won't be able to distinguish situations in which your beliefs were vague from your choices, from the resulting probability distribution. Ax If prob dist had very low variance--if you acted "as if you were almost certain that 1 or 2 events out of n having some probability would occur--e.g. be willing to give high odds that it would occur and demanding high odds to bet against it--we could infer that your beliefs were not vague. But if you obey the axioms (e.g., if you took the midpoint of an interval of probs as the value), we couldn't infer that a prob dist of high variance represented vaguer beliefs than one of low variance. R &S: we can talk about the xx effect a small number of observations would have on your beliefs, but one probability distribution is as "definite" as another (though there may be more inconsistency, e.g., violation of P1 or change of choices from trial to trial)xunder some choices than others); can't say a uniform dist. is less definite.

3. Notion of vagueness: you can't judge whether one proposition has a higher degree of belief than another; you don't know your own mind, you can't say, you aren't sure (a subjective reaction to two sorts of situation: a) poverty of cues, hazy, unstructured, unfamiliar; b) lots of cues, but contradictory and in background of noise, overload of prominent indications, hard to make out pattern (Keynes: if the clouds are black but the barometer is fair). Ambiguous.

4. Can assume responses to this will differ with personality; tolerance/intolerance of ambiguity, willingness to recognize it in one's own state of mind, nature of perception, response if recognized, or not. If situation is important: wishfulness, picking out cues that are favorable (may have definite probabilities; may not recognize ambiguity).

5. But how to make inferences from vague beliefs; how to describe them, compare them, measure them, measure range of vagueness; how to arrive at decisions despite vagueness, take actions that will be consistent with what knowledge you do have.

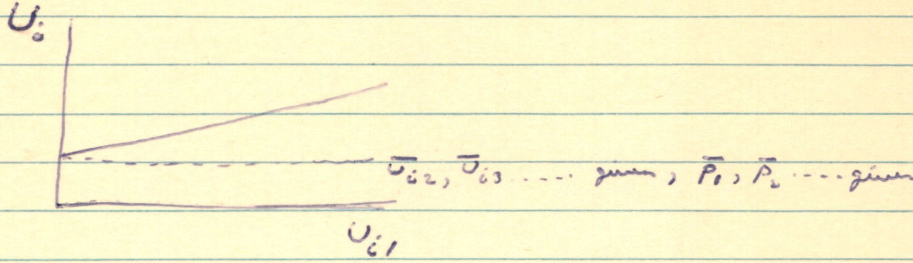
6. We may know utility payoffs, alternatives, relevant events; we know how we would act if we had ~~[illegible]~~ definite beliefs; "if we felt that E was more probable than F..." "if we thought E were more probable than 1/10...

It is comparisons of this sort we need for decision; but Ramsey-Savage approach proceeds to specify conditions for determing numerical probability exactly. ~~We want~~ Let us ask instead: how can we establish meaningfully that prob(E) is greater than 1/10?

7.

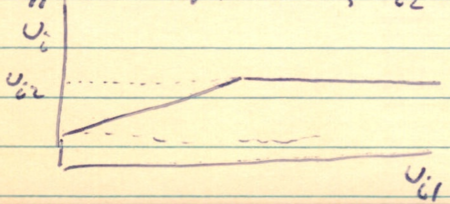
Value of Intelligence

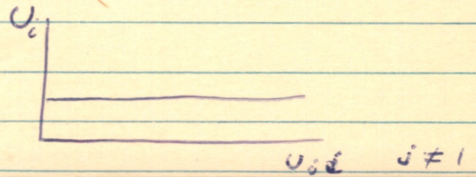
Under risk, the *improvement* in utility of an action i is *proportional* to the improvement in any one of its outcomes U_{ij}



Under ambiguity, say $\rho = 0$, $\alpha = 0$ (pure minimax), improvement in U_i (index value) is non-linear function of improvement in U_{ij}: 0 if U_{ij} is not *min* for ~~[illegible]~~ i; proportional for $U_{ij}^{\underline{min}}$, until that outcome reaches level of *next lowest* outcome, then 0.

Let U_{i1} be min for ~~[illegible]~~ i, U_{i2} be next lowest outcome.





over

So index for a *set* of actions will be *sensitive* only to improvements in certain minima; a) any improvement in the highest minimum; b) an improvement in another minimum sufficient to make *it* the highest.

Get: Cowles Foundation Discussion Paper 46: Bargaining in ignorance of the opponent's utility function; john Harsanyi, Dec. 11, 1957

No. 48: On incentives to forecasters and to decision makers under uncertainty Harsanyi, Jan. 10, 1958

59: Team theory, Marschak and Radner:
31: Theory of Teams, introduction: Marschak
59 c:
59d
59e

Marschak notes to Economic 115a, 116b

Bayes measured probs by wagers ("bets on singular events were common in England, lotteries--leading to interest in repetitive, frequency phenomena, on the Continent") assuming utility linear on money.

Ramsay: Axiom 1 implies that person's judgment about prob of event is independent of rewards attached to its occurrence.

Ramsay assumes that individual acts according to a set of numbers describing his tastes and a set of numbers describing his beliefs, both sets being independent of each other; he will choose among wagers as if he were maximizing expected utility. (he does not assume this directly, but derives it from more plausible axioms, though these together are no less strong than the theorem).

Where wagers involve only outcomes a and b, instead of saying that individual maximizes expected utility we can say that he maximizes the probability of gain. Savage:

P1, assuming complete ordering of acts, + of events gives transitivity of ordering of events ~~probabilities~~; Def: If $b > a$ and $bEa \geq bFa$, then E is at least as probable as F: $E \geq F$.

P1: $bEa \geq bFa$, $bFa \geq bGa \Rightarrow bEa \geq bGa$. Hence $E \geq F$, $F \geq G \Rightarrow E \geq G$.

~~Px~~ P4 insures comparability of event-weights ~~probabilities~~, ruling out possibility that:
$bEa \geq bFa$ but $cEd < cFd$. P4: For all E, F, either $E \geq F$ or $F \geq E$.

P1. and P4 give a complete ordering on the set of events (P1 alone would only give partial ordering on events). To get qualitative probabilities, we must have a weak additivity condition, implying that the sum of A and B cannot be less than A; i.e., that if A is more likely than C, A and B cannot be less likely~~x~~ than C and B (where C is disjoint with A and with B). If $E \geq F$ and $EG = FG = 0$, then $(E \text{ or } G) \geq (F \text{ or } G)$

This additivity property follows from Sure-thing Principle, which Marschak renders: along with P1 (+ P4?)
If $b > a$, then $bEx > aEx$ for all x.

Corresponds to Wald's "admissibility principle" in statistical decision theory, and rule of "dominant strategies" in theory of games. The sure-thing principle in itself establishes only a partial ordering on the set of wagers.

Additivity property in terms of wagers:

If $bEa \geq bFa$ and $EG = FG = 0$

then $b(E \text{ or } G)a \geq b(F \text{ or } G)a$.

To get numerical probs: If $E \geq F$, then $\exists$ event $G \ni E \geq (F \text{ or } G)$

Parametric hypotheses + Edwards hypothesis inconsistent with expected utility axioms.

2.

Savage derives at once the whole probability scale, using only two outcomes.; then proves the existence of a subjective utility function, like vn-M. Ramsay defines subjective probability $=\frac{1}{2}$ in same fashion, then measures equidistant utilities, derives a numerical utility scale, then comes back to measure probabilities according to Bayes' definition, substituting utilities for money.

In Savage approach, we don't ask how a person's beliefs were formed, whether or not they were related to past experience and whether or not they were being revised in the light of new experience. But by measuring his joint probabilities and conditional subjective probabilities, Bayes' theorem states how his probabilities should be affected by observational experience:

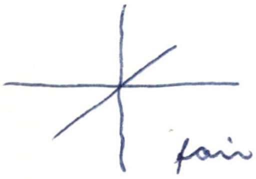
The a posteriori probability of a hypothesis is proportional to its a priori probability and its likelihood, given the observation.

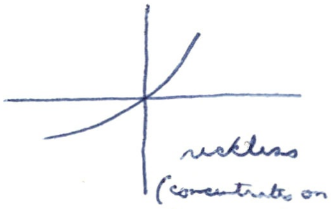
The ratio of the probabilities of H' and H after r has been observed is equal to the old ratio, corrected by the factor ~~p(r/H)~~) $\frac{P(r|\bar{H})}{P(r|H)}$ = ratio of "likelihoods" of $\bar{H}$ and H.

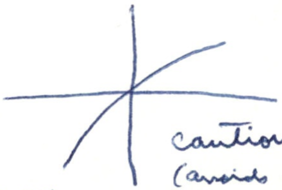
$$\frac{P(\bar{H}|r)}{P(H|r)} = \frac{P(r|\bar{H})}{P(r|H)} \cdot \frac{P(\bar{H})}{P(H}$$

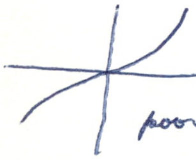
With increasing number of observations, the subjective probability assigned to the false hypothesis H' approaches 0.

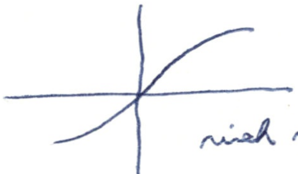
Shape of utility fn. of wealth: See Kemeny, Contributions to Game Theory, Vol III

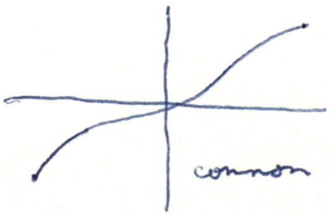

fair

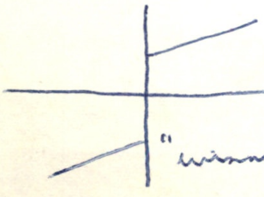

reckless
(concentrates on large gains, discounts large losses).

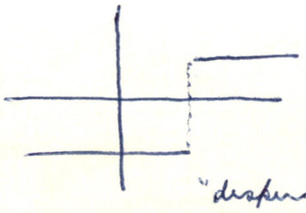

cautious
(avoids large losses, discounts large gains).

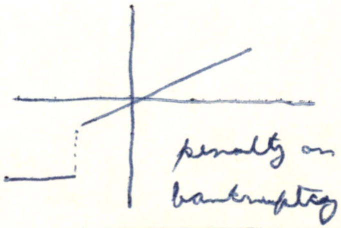

poor man
(overrates large gains and large losses)


rich man
(underrates large gains and large losses)


common
(reckless for small amounts, cautious for large)


"winning"
+ premium on winning, negative on losing


"desperate"
(level of aspiration)
2-valued utilities
DETERRENCE: SPASM WAR


penalty on bankruptcy

Distinguish between: cautious optimist*; reckless optimist; cautious pessimist; reckless pessimist
(* only one to diversify;

Also, if we attribute the same crude payoff structure to the opponent (0,1) th exaggerates the identify of interest between us and the opponent, in this case, and permits him to steal our shirt, if he doesn't assign same payoffs, without encouraging him to cooperate in measures that really reduce likelihood of war. CONVERSELY, THOSE WHO ASSIGN -1,1 payoffs to "VICTORY" AND "DEFEAT"--and attribute 1, -1 payoffs to the enemy--exaggerate the amount of conflict; and ignore measures that reduce the likelihood of mutual disaster, or that improve the chance of an "acceptable outcome" at the cost of the chance of "total victory."

37. "Can we afford to be conservative?" Suppose both the best guess and the worst distribution for I are (slightly) better than for II, which is also more ambiguous; but all of these expectations are "terrible," "disastrous," "below the level of aspiration"; and II offers a better "best" expectation, in fact the only possibility of attaining the level of aspiriation, a viable or acceptable or non-disastrous expectation. How would we explain formally the--rather plausible--choice of II in this case?

0,1

PAYOFFS

If the "best" expectation, corresponding to II is only "slightly better" than the best or the best guess corresponding to I, then II would be chosen only if is quite large: the person is typically "optimistic" or "wishful" under ambiguity. This sort of case is an argument for α being greater than 0; if α =0, you could never give any weight to optimistic possibilities.

Suppose $\alpha >$ 0, but less than ½. This could lead to the choice of II if the "best" is much better than the other expectations; and this is really implied by the description of the situation. In fact, those who felt that the "best" distribution is enough better to justify II (these people may honestly, or ignorantly, underestimate the differences between the other expectations) may argue in strict level-of-aspiration terms, that the differences between other possible outcomes are negligible, that they are all terrible, that the payoffs can be represented by 0 and 1, so that the problem becomes one simply of maximizing the probability of the payoff 1, disregarding differences in the probabilities of objective outcomes other than this "best" outcome, and accepting no reduction in the probability of the best payoff for the purpose of reducing the probability of the worst, or raising the probability of some intermediate outcome.

This approximation may be more realistic, and may lead to "better" decision-making, than one which exaggerates relative differences between various outcomes less than the best, without focussing clearly on the enormous gap between the best of these and the "best" (or, the least of the "acceptable" set of outcomes) or on the generally"unacceptable" nature of these outcomes. (The latter perception should not necessarily lead to choosing that currently perceived strategy which offers maximum chance--which may be very small--of an "acceptable" outcome; a better response might be increased search for new alternatives, and efforts to change the likelihoods of contingencies).

However, this approximation may be extremely crude and lead to ignoring disastrously differences in objective outcome which really correspond to vastly different payoffs. The problem is not to exaggerate the difference between 40 million dead and 100 million dead (relative to the difference between 40 million dead and 0 dead but political setback or negotiated settlement), but at the same time not to ignore it, to act "as if" they were really identical in our values.

SANE regards any state of the world involving "0 likelihood of nuclear war" as having payoff 1, any state with any positive likelihood of nuclear war as having payoff 0. Thus they prefer any strategy that offers any possibility of achieving a payoff of 1 ("general and complete disarmament"), though combined with the possibility of states with high potential for nuclear war, to any strategy which does not offer the possibility of 0 potential for nuclear war but which lowers drastically the probability of states with high potential for war, and which particularly lowers the probability of the most destructive forms of war.

SANE may argue that RAND focuses too much upon the latter sorts of "gains"; that RAND doesn't spend enough time searching for new strategies with drastically better expectations; or even (unfairly) that RAND is willing to accept increases in the likelihood of in order to improve the outcome; or even that RAND prefers states with a possibility of war to ones without (libelous). Yet in seeking the impossibility of war, SANE may increase the probability of war, and of disastrous war, ~~and~~ or fail to decrease it; or to improve the outcome without affecting the likelihood.

1) What is the ratio p_* such that $v(I) = v(II)$?

I	S	0
II	pS	pS

2) What is the ratio q_* such that $v(III) = v(IV)$?

III	0	S	$-S$
IV	qS	qS	

0 1 0 $\frac{1}{6}$

1 2 1 1 4

3) What is the relation ~~of~~ between q_* and p_* required by coherence?

$$p_* \leq 1 - q_*, \text{ or } p_* + q_* \leq 1$$

For suppose $p_* + q_* > 1$:

Person would be willing to give up p_*S for the wager I; and
he " " " " " q_*S " " III.

Thus, he would pay $p_*S + q_*S > S$ for the combined "wager"

E	$\bar{E}$
S	S

, or the certainty of S; he would be certain to lose $(p_* + q_* - 1)S$, no matter what happened.

But suppose $p_* + q_* < 1$; no S can be found for which a book can be made against him; he doesn't violate this condition, whatever else he may violate.

2 1 0

$$\left(\begin{array}{cccc} E & & E & \\ 1 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 \\ -1 & 1 & -2 & 1 \end{array}\right)$$

$x_\alpha \leqq 1 - y_\beta$

$y_\alpha \geqq 1 - x_\beta$

$1 - y_\beta + 1 - x_\beta = \frac{2}{3} = x_\beta + y_\beta$

-2 4

$y_\beta =$

$p \cdot 2 + (1-p)(-1) = 2p - 1 + p = 0$ $p = \frac{1}{3}$

$-2p + (1-p)(1) = -2p + 1 - p = 0$ $p = \frac{1}{3}$

$3p - 1 \leq 0 = -3p + 1$

2 -1

~~p~~ 0 0

$3p - 1 < 0$ $3p < 1$

$-3p + 1 < 0$ $3p > 1$

$x_\alpha + y_\alpha = \frac{2}{3} = x_\beta + y_\beta$

-2 1

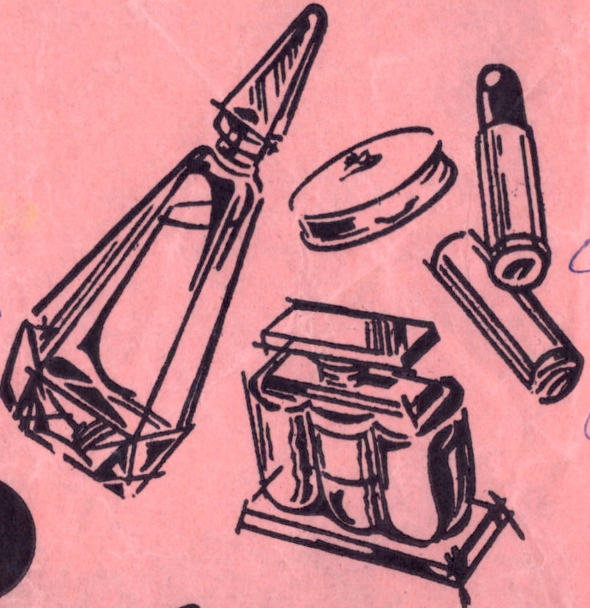
0 0

$x_\alpha - x_\beta = y_\beta - y_\alpha$ 3 0 1 0 0 1 0 3

$x_\alpha > x_\beta$ and $y_\beta > y_\alpha$ 1 1 $\frac{1}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{2}{3}$ 2 2

$\frac{2}{3} - x$

$x \geq 1 - \frac{2}{3} + x = \frac{1}{3} + x$



$x \geq 1 - \left(\frac{2}{3} - x\right) = \frac{1}{3} + x$

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Emile Borel: "A Propos d'un Traite de Probabilites," Revue Philosophique, t 98, 1924, 321-336. Valeur Pratique des Probabilites, Gauthiers-Villars, Paris, 1939.

il est des cas ou il est legitime de parler de la probabilite d'un evenement; ce sont les cas ou l'on pense a la probabilite commune aux judgments de tous les hommes les mieux informes, c'est-a-dire possedant tous lesrenseignements qu'il est humainementpossible de posseder a l'hueure actuelle. Sans doute, cette probabilite pourraetre modifiee par une nouvelle decouverte scientifique, mais c'est aussi le cas pour beaucoup de donnees numeriques que l'on appelle cependant des constantes physiques et chimiques; laprobabilite qu'un atome de readium explose dans la journeee de demain est, pour le physicien, une constante au meme titre que la densite du cuivre ou le poids atomique de l'or; ces constantes sont toutes a la merci d'un progres de la theorie physico-chimique; ce sont cependant des constantes dans l'etat actuel de la science. 137

Les probabilites subjectives, au contraire, qui paraissent interesser particulierement M. Keynes, peuvent avoir effectivement des valeurs differentes suivant les individus; tel est le cas, par exemple, pour leshabituesdescourses de chevaux, dont chacun cherche a supputer les chances des partants d'une course in tenant compte des divers renseignements qu'il peut posseder sur tous ces partants. On a dit parfois que, sur deux parieurs, il y a toujours un voleur et un imbecile; cela est vrai dans certains ca, ou l'un des deux parieurs est beaucoup mieux renseigne que l'autre, et le sait; mais il peut fort bien arriver que deux hommes de bonne foi, dans des matieres complexes ou ils possedent exactement les memes elements d'information, arrivent a des conclusions differentes sur les probabilites d'un evenement et que pariant ensemble, chacun se figure, pour employer ces expressions pittoresques mais excessives, que c'est lui le voleur et l'autre l'imbecile.
137

De meme lestemperatures peuvent etre definies et mesurees avec une precision de plus enplus grande par les physiciens. Au contraire, la sensation subjective de chaud et de froid ne serait pas sans analogie avec la probabilite telle que l'entend M. Keynes. Lorsque le thermometre marque 2°, deux hommes differents, ou un meme homme suivant les circonstances, pourront employer des expressions fort differentes pour apprecier la temperature. Un physicienconsidere neanmoins la notion de termperature comm une notionscientifique precise, susceptible d'une definitkon rigoureuse qui en permet l'evaluation numerique exacte. En est-il de meme pour la notion de probabilite ou doit-on reconnaitre, avec M.Keynes, qu'il est des probabilites ne pouvant etre evaluees numeriquement? 139

Dans bien des question pratiques de probabilite, on est fort loin de pouvoir esperer meme 3 ou 4 decimales exactes; il suffit que la probabilite soit comprise entre certaines limites connues pour qu'il soit legitime de parler de sa valeur numerique et de l'introduire dans les calculs. Bien entendu, il est desirable que ces limites soient aussi resserrees que possible, mais c'est la une question de plus ou de moins, qui ne touche pas aux principes; tout nombre introduit dans les calculs pratiques est seulement connu d'une maniere plus ou moins approchee; il en est des probabilites comme des autres nombres.

(que l'on ait 5 decimales exactes ou que l'on en ait 10, le nombre des decimales inconnues rest aussi grand.) 139

(((THISISTRUE OF ALL MEASUREMENTS: BUT WHEN WE CONSIDER HOWTO ACT ON THE BASIS OF APPROXIMATE MEASUREMENTS...OUR OPINIONS AS TO THE "TRUE" VALUE MAY BE DEFINITE: OR THEY MAY NOT...))

Il faudra reconnaitre cependant qu'il y aura rarement avantage a soumettre au calcul des nombres trop imparfaitement connus. Si je veux evaluer la surface d'un champ et si je me contente, pour evaluer ses dimensions, de le regarder rapidement en passant en automobile sur la route qui le bourde, il y aurades chancespour que mon evaluation soit tres inexacte et le prix que j'en deduirais ne serapas moins exacte. De meme, si l'on medemande quel sera le gagnant dans une course de chevaux alors que j'ignore tout des partants, la probabilite de ma reponse n'aura aucune valeur precise, ni a mes yeux, ni aux yeux d'un interlocuteur aussi mal renseigne que moi. Si, cependant, sans rien savoir de precise au sujet des courses

$pr(E) = \frac{1}{3}$
and
$pr\,\bar{E} = \frac{2}{3}$

$$\begin{array}{cc} E & \tilde{E} \\ \textcircled{\tfrac{2}{3}} & -\frac{1}{3} \\ 0 & 0 \\ -\frac{2}{3} & \frac{1}{3} \end{array}$$

← assumption that you can multiply this by −1 without affecting its preference relation to 0 0 (not betting) amounts to assumption ... —

Can multiply, since utilities:

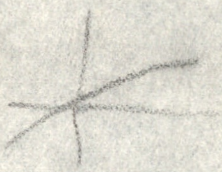
$$\begin{array}{ccc} & A & \\ & ② & -1 \\ \times(-1) & 0 & 0 \\ \textcircled{B} & \textcircled{-2} & 1 \end{array} \quad \sim \times\tfrac{1}{2} \quad \begin{array}{ccc} r & & \\ ② & -1 & \\ 0 & 0 & \\ -1 & \textcircled{\tfrac{1}{2}} & R \end{array}$$

A

$A \geq -B$?

$r \geq \frac{1}{R}$

A person who is "sure to lose utility" isn't sure to lose money, if his utility is convex (?).



If he is also "conservative," then a) by utility: he will accept only bets whose expectation of money is strongly positive $E(m) > \epsilon > 0$, even when probs are "unambiguous"

b) When probs are ambiguous, $E(m)$ must be even more "favorable"; in fact, $E(\underline{U})$ must be strongly "favorable" (in terms of his "best guess" probs) to a degree depending on ρ, α, and X_{min} and X_{max}.

Do I assume set of "possible" dists is convex? If so, he ends up with a "possible" dist. (Yes, if they are ~~the~~ "not unreasonable" dists.

Est:

α is the g.l.b. of a's + b's ∋

$$\begin{array}{cc} E & \tilde{E} \\ 1 & 0 \\ a & a \end{array} \qquad \begin{array}{cc} E & \tilde{E} \\ 0 & 1 \\ b & b \end{array}$$

over ~~all~~ many ~~events~~

ambiguous events (for which $a + \frac{1}{b} < 1$).

~~[illegible]~~. g.l.b. of max(a, b)

---

for $r_i \geq \frac{1}{2}$, $p_i \geq r_i$

If $\alpha = 0$; $r_i \leq p_i \leq (1-R_i)$ (~~[illegible]~~ if p_i were $> (1-R_i)$

$R_i > 1 - p_i = q_i$; I wouldn't be willing to pay R_i for $\begin{array}{cc} E & \hat{E} \\ 0 & 1 \end{array}$.),

If ~~[illegible]~~ $0 < \alpha < \frac{1}{2}$, then for p_i small ($< \frac{1}{2}$), it may be that

$$p_i < r_i < (1-R_i) < (1-p_i)$$

Two modes; when payoffs are small, $\rho = 1$.

Like Preston + Baratta results (though explain this by l.m.d. > 5-10¢, convex utility over 50¢ ?)

$$\begin{array}{ccc} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{array}$$

$$\begin{array}{ccc} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{array}$$

	$\frac{1}{3}$	$0-\frac{2}{3}$	$0-\frac{2}{3}$
I	1	0	0
II	0	1	0
III	0	0	1
IV	1	1	0
V	1	0	1
VI	0	1	1
VII	1	1	1

$$\begin{array}{ccc} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \end{array}$$

$\| \quad 0$

VII $\quad \frac{1}{3}$

$P_1 > P_2$

VI

IV – V $\quad \frac{1}{3} > P_3$

I

II – III $\quad \frac{1}{3} < P_2 + P_3 \Rightarrow P_2 > \frac{1}{6}$

$\Rightarrow P_3 > \frac{1}{6}$

$\frac{1}{3} + P_2 < P_2 + P_3$

For any pair of actions, I, II, $\exists C \ni$ either

$I_p II \ \forall c \geq C$ or $I_p II \ \forall c \leq C$.

0

$\frac{2}{3}, \ \frac{2}{3}C$

$$\begin{array}{ccccc} 1 & 0 & \frac{1}{2} & & \frac{1}{3} \\ 0 & 1 & \frac{1}{2} & & \frac{1}{3} \end{array}$$

$$\begin{array}{ccccc} 1 & 0 & \frac{1}{4} & & \frac{1}{3} \\ 0 & 1 & \frac{1}{4} & & \frac{1}{6} \end{array}$$

Axiom: If $I_p II$ for some c, then either

$I_p II$ for all $c' \leq c$ or $I_p II$ for all $c' \geq c$.

or: either $I_p II$ for all $c \geq a$

If $I_p II$ for $c \geq a$, then $I_p II$ for all $c' \geq a$

If $I_p II$ for $c \leq b$, then $I_p II$ for all $c' \leq b$.

Math Rev I · 1946

Bull Amer Math Soc 46 1940

$\frac{2}{3}$

$$\left.\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right| \begin{array}{c} \frac{1}{3} \\ 0 \end{array} \left| \begin{array}{c} \frac{1}{3} \\ \frac{2}{3} \end{array}\right.$$

III

$$\left.\begin{array}{ccc} 1 & 0 & -10 \\ 0 & 1 & -10 \end{array}\right| \begin{array}{c} -\frac{19}{3} \\ -\frac{20}{3} \end{array} \left| \begin{array}{c} \frac{1}{3} \\ \frac{2}{3} \end{array}\right.$$

$$\begin{array}{ccccc} 1 & 0 & 0 & & \frac{1}{3} \\ 0 & 1 & 0 & & 0 \end{array}$$

$$\begin{array}{ccc} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \end{array}$$

$$\boxed{\begin{array}{c} \left.\begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right| \begin{array}{c} \frac{1}{3} \\ \frac{2}{3} \end{array} \left| \begin{array}{c} 1 \\ \frac{2}{3} \end{array}\right. \\ \\ \left.\begin{array}{ccc} 1 & 0 & 10 \\ 0 & 10 & 10 \end{array}\right| \begin{array}{c} \frac{1}{3} \\ \frac{2}{3} \end{array} \left| \begin{array}{c} \frac{21}{3} \\ \frac{20}{3} \end{array}\right. \\ 60 \end{array}}$$

$$\begin{array}{ccc} 1 & 0 & c \\ 0 & 1 & c \end{array}$$

$\frac{1}{3} \longrightarrow \frac{1}{3} + \frac{2}{3}c$

$\frac{2}{3} - \frac{2}{3}c$

$$\begin{array}{cccccccccc} & 1 & 0 & 2 & \frac{1}{3} & \rightarrow & 1 & 0 & -1 & -\frac{1}{3} \\ \rightarrow & 0 & 1 & 2 & \frac{2}{3} & & 0 & 1 & -1 & -\frac{2}{3} \\ & 1 & 0 & & & & & & & \end{array}$$

$$\begin{array}{ccc} a & b & b \\ b & a & b \\ a & b & a \\ b & a & a \end{array}$$

1

2

RAND thinks US should buy posture with "insurance" against contingencies other than the "most likely" attack.

But gov typically a) won't insure unless other contingencies are *quite likely*; b) regards these other contingencies as *very unlikely*.

Other contingencies in question include SU "well-designed attack" – reflecting physical constraints but not doctrinal/political/traditional.

Reason for (a) is partly that "insurance" is regarded as having very high cost (budget, inflation, etc). (b) reflects lack of imagination or expertise, wishfulness, stereotype of enemy.

So RAND has to (1) focus on that case, explain it, make it seem "possible"; (2) temptation to exaggerate its likelihood.

Insistence on "point estimates" will lead to greater pressure to bias prediction conservatively, to force you to "buy insurance" unwittingly. (This leads to predictions unduly likely to be *disproved*. Alsop.)

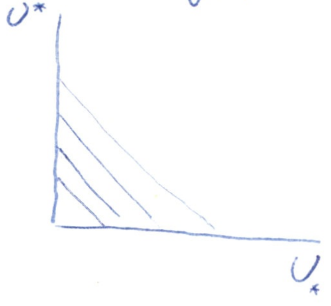
Point Predictions should often be regarded as ~~statements~~ imperatives: "Act *as if* the likelihood were -----"; the predictor having specific actions in mind.

Estimate of range of reasonable distributions could be made *independent* of actions/payoffs.

See Churchill, The Grand Alliance, pp 38-41 α

Suppose my preferences over (U_*, U^*) depend not only on $|U_* - U^*|$ but on the "levels" of U_* and/or U^*.

~~Shackle~~ hyp: Hurwicz hyp:



Shackle

α ~~increasing~~ decreasing fn. of U_*^*
~~≡~~ in problem

I	1	0	0	1	0
II	1	0	1	1	0

$p^*(\bar{E}) \leq p_*(\bar{F})$

$(1-p_*(E)) \leq (1-p^*(F))$

$p_*(E) \geq p^*(F)$

$P_*(E) \geq p_*(F)$

and $p_*(\bar{E}) \leq p_*(\bar{F})$

$(1-p^*(E)) \leq (1-p^*(F))$

$p^*(E) \geq p^*(F)$ necessary

$p_*(\bar{E}) = (1-p^*(E)) \leq p_*(F)$

or $p^*(E) \geq (1- p_*(\bar{F}) = p^*(F)$

p^*

b a a a

$$\left(\begin{array}{cccc} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{array}\right.$$

$p_*(\bar{E}) \geq p_*(\bar{F})$

$\frac{1}{3}$ $0-\frac{2}{3}$

$p^*(\bar{E}) \gtrsim p_*(F)$

~~$P_*(E)$~~

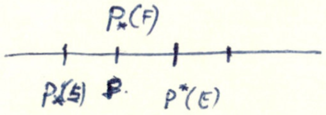
$p^*(\bar{E}) \geq p_*(\bar{F})$

$(1-p_*(E)) \geq (1-p^*(F))$

$P_* \leq P^*(F)$

$(1-p_*(E) \geq (1-p^*(F))$

$P_*(E) \leq p^*(F)$



1. Heuristic assumption. Results....

2. To act reasonably is, above all, to judge actions by their consequences. But what of their consequences are unknown? One ~~should~~ must still, no doubt, act reasonably; only few ~~[illegible]~~ believe they know that means, and those who do disagree.

3. Can ~~assume~~ ignore uncertainty, or assume that decision-makers

4. Lack a language for talking about uncertainty, meaningfully and clearly; useful distinctions; formalizations... Prior to: how do people act?

5. Normative theory as prolegomena (Churchman).

6. Yet people must decide; turn to econs for advice. Reverse heuristic: learn what to do from what people do.

7. But statisticians reawakened interest; provide model, decision rules, (counter-examples) axioms.

8. Various decision

$\frac{1}{2}\cdot\frac{1}{3} + \frac{[illegible]}{3}$ $\quad \frac{1}{\ }$

$\frac{1}{2}\Big[.4\cdot\frac{2}{3} + .60\Big)$ $\quad .2$

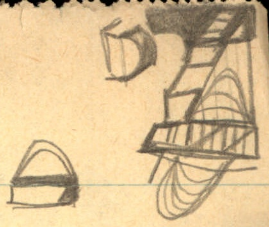
D-1114

$\rho\cdot y_0 + (1-\rho)\big[\alpha\cdot y_{max} + (1-\alpha)y_{min}\big]$

$\left(\frac{1}{3}, 0, \frac{2}{3}\right)\left(\frac{1}{3}, \frac{2}{3}, 0\right)$

R	Y	B
		0
1	0	0
0	1	

	R	Y	B	
I	1	0	0	$\frac{1}{3}$
II	0	1	0	
III'	0	0	1	
III	1	1	0	
IV	0	1	1	
IV'	1		1	



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$\frac{1}{2}\cdot\frac{1}{3} +$ [illegible]

$\frac{1}{2}\left[.4\cdot\frac{2}{3} + .60\right)$ $.2$

D-1114

$\rho\cdot y_0 + (1-\rho)\left[\alpha\cdot y_{max} + (1-\alpha)y_{min}\right]$

$(\frac{1}{3}, 0, \frac{2}{3})$ $(\frac{1}{3}, \frac{2}{3}, 0)$

R	Y	B
		0
1	0	0
0	1	

	R	Y	B	
I	1	0	0	$\frac{1}{3}$
II	0	1	0	
III'	0	0	1	
III	1	1	0	
IV	0	1	1	
IV'	0	[illegible]	1	

0 0 2 -1 0 0 -2 1
0 0 0 0 0 0 0 0

---

0 0 -2 1 0 0 0 0
0 0 0 0 0 0 -2 1

0 0 0 0
0 0 2 -1

-2 -2 +2 +2 P1
0 0 2 -1

-2 -2 -1 2 P2
0 0 -1 -1

-2 -2 -0 2 P1
-1 -1 0 0

0 0 0 0
0 0 2 -1

1 1 -1 -1 P1
0 0 2 -1

1 1 -1 2 P2
0 0 2 2

1 1 -1 2 P1
1 1 1 1

-1 -1 -1 2 P2
-1 -1 1 1

-2 -2 -2 2
0 0 -2 -1

-1 -1 -1 2 P1
0 0 0 0

-1 -1 -1 -2
0 0 -2 1

	B	C
Bet	x	$-wx$
Reject	0	0

	B	C
	1	$-w$
	0	0

$w = w(B:C)$ is l.u.b. w such that Bet ≥ Reject.

odds on B against C

	B	C
I ~~Bet~~	1	0
II ~~Reject~~	$\bar{p}_*$	$\bar{p}_*$

p_* is l.u.b. of payments for bet, i.e. p's $\ni$ I ≥ II,

or $\ni$ $\begin{array}{ccc} I' & 1-p & -p \\ II' & 0 & 0 \end{array}$ $I' \geq II'$.

$$\begin{array}{ccc} I'' & \frac{1-p}{p} & -1 \\ II'' & 0 & 0 \end{array} \qquad \begin{array}{ccc} I' & 1-p & -p \\ II' & 0 & 0 \end{array}$$

$$I' \geq II' \Leftrightarrow I'' \geq II''$$

II' is bet on B at $\frac{1-p}{p} : 1$ on B.

$w = w(B:C)$ are lower odds on B against C.

i.e. if I must have at least 3:1 on E $\begin{array}{cc} 3^+ & -1 \\ 0 & 0 \end{array}$ to bet, I should be willing to pay at most $\frac{1}{4}$ for I; this is lower prob for E.

Suppose $p_*\cancel{(S)} > p^*\cancel{(-S)}$

	E	$\bar{E}$
I	$(1-p_*)S$	$-p_*S$
I′	$-S(1-p^*)$	p^*S
II	0	0

By def of p_* and p^*; I will accept both I and I′

(both indifferent to II)

I″	E	$\bar{E}$
	$(1-p_*)S - S(1-p^*)$	$-p_*S + p^*S$
	$\cancel{S} - p_*S - \cancel{S} + p^*S$	$S(p^* - p_*)$
	$S(p^* - p_*)$	

$$p^* - p_* \geq 0$$

4) Let p^* be that p for which $v(\underline{V}) = v(\underline{VI})$.

$$\begin{array}{lll} \underline{V} & -S & 0 \\ \underline{VI} & p^*(-S) & p^*(-S) \end{array}$$

$v(\underline{V}) = v(\underline{III}) - S$

Hence, $p^*(-S) = q_*(S) - S = -S(1-q_*)$

Thus, $p^* = 1 - q_*$

$p_0 = 1 - q_0$

By condition of coherence, $p_* \leq p^*$

(Show)

5) de Finetti ...... all (except Koopman, Smith) assume that

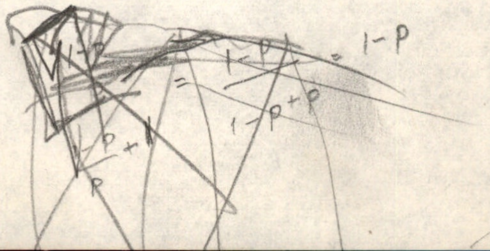
$p_* = p^*$. This is required if p is to be numerical and well-defined without reference to S, ~~S either~~ equivalent for both S and -S for any S.

But it is not required by condition of coherence alone.

e.g. since most gambling houses successfully make book against the players, there is ~~no way to prove~~ no indication that their "beliefs" are definite in this sense; though they are clearly coherent.

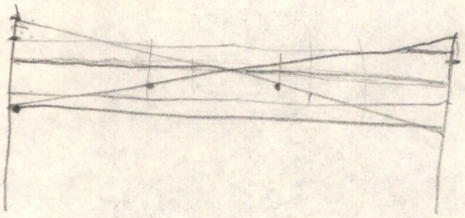
$W_0 = \frac{1-p}{p}$ $\quad pW_0 = 1-p$ $\quad \frac{W_0}{1+W_0}$

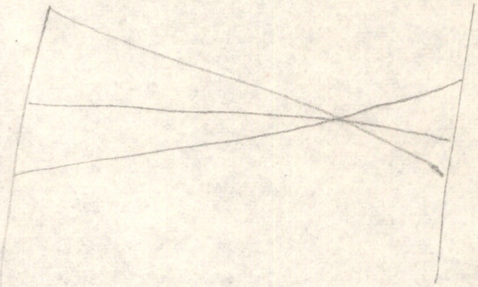
$p(1+W_0) = 1$ $\quad p = \frac{1}{1+W_0}$



$= \frac{1-p}{1-p+p} = 1-p$

$\frac{1-p}{p} + 1$





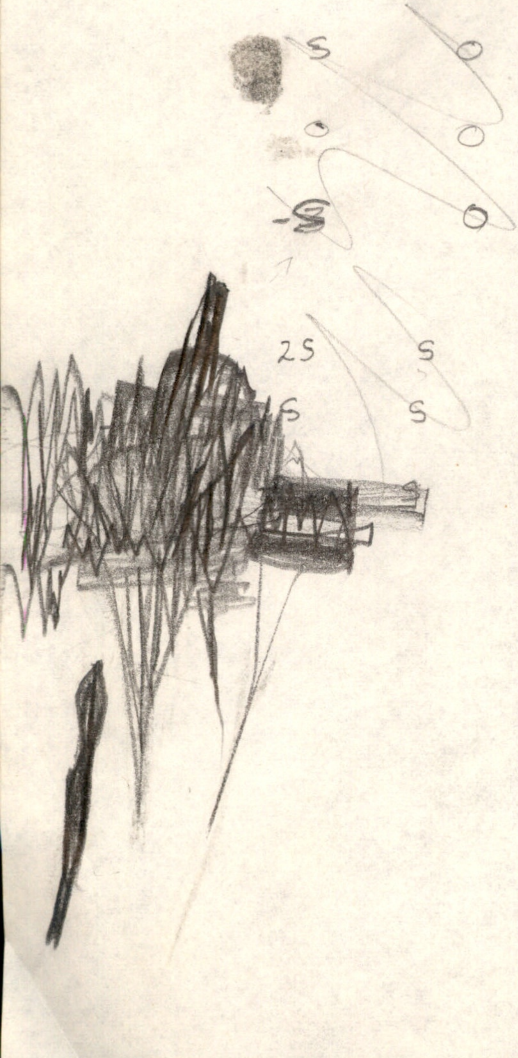
I	S	O
II	O	S
III	$\frac{S}{2}$	$\frac{S}{2}$

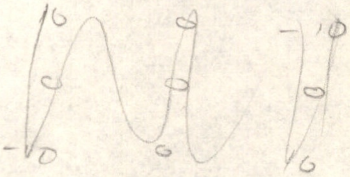
H	T
$\frac{S}{2}$	$\frac{S}{2}$
S	O
O	S

W	-1
O	0
-W	1

Can't draw I < III, II ≤ III by axioms.

Can't *prefer* to toss a coin to *both* of I, II.





P-2179-RC

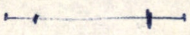
~~1 1 p p~~

$$\left(\begin{array}{cccc} 1 & 0 & p & p \\ p & p & p & p \end{array}\right.$$

$$\left(\begin{array}{cccc} 0 & 1 & q & q \\ q & q & q & q \end{array}\right.$$

$$\begin{array}{cccc} \hline 1 & 0 & 0 & 0 \\ p & p & 0 & 0 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 0 & p & p \\ \hline 1 & 1 & 0 & 0 \\ 0 & 1 & p & p \end{array}$$

$$\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ & & & \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{array}$$



$$\frac{1}{2}\cdot\frac{1}{3}+\frac{1}{2}\cdot 0=\frac{1}{6}$$

$$\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{3}$$

$$-\frac{1}{3} \qquad \frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{2}$$

$$\left(\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right.$$

$$\left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ p & p & 0 & 0 \end{array}\right.$$

$$\left(\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1-p & 1-p & 0 & 0 \end{array}\right.$$

$$\left(\begin{array}{cccc} 0 & 0 & 1 & 0 \\ 0 & 0 & p & p \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1-p & 1-p \end{array}\right) \text{then}$$

Certain risk-combinations of ambiguous gambles can eliminate *ambiguity as to the payoffs* (which is the important ambiguity); the result is a lottery ticket preferred to any of its components.

Test of ~~an~~ a normative axiom is not whether it "looks reasonable," or whether we *think* we want to conform to it, but whether we *do* want to conform to it, when tested (Savage assents).

I would have predicted that I wanted to conform to Sure-thing, but I *find* that in certain cases, I don't.

Discuss purpose of normative theory.

Pragmatic justification (for most cases) — like heuristic rule, when your goal is discovery — rule of thumb

justify use of best + worst (all *events* and all *payoffs* are represented; ~~but~~ unlike minimax rule or Hurwicz — but not all distributions, terms of trade, weights). Prominence; ease of computation; *adequate* pragmatically (for S) and descriptively (for O).

"When people (Knight, Savage, Keynes) *say* they don't *allow* for optimism, confidence, etc., they mean that their theories have the following characteristics . . . they treat the following situations as 'identical . . ."